PLANE-STRESS YIELD CRITERION FOR HIGHLY-ANISOTROPIC SHEET METALS

Dan-Sorin Comsa, Dorel Banabic*

Technical University of Cluj-Napoca, 15 C. Daicoviciu, 400020 Cluj-Napoca, Romania

ABSTRACT: The paper presents a plane-stress yield criterion in the form of a finite series that can be expanded to retain more or less terms, depending on the volume of experimental data. Due to its structure, the model is suitable for a variety of practical applications. An identification procedure consisting in the minimization of an error-function is used to evaluate the coefficients included in the yield criterion. The effectiveness of this strategy is proved for the particular situations when sets of 8 and 16 experimental values are available. In both cases, the input quantities (normalized yield stresses and r-coefficients) are obtained from uniaxial and biaxial tensile tests. The performances of the yield criterion are evaluated by comparing its predictions with the experimental data for an AA2090-T3 aluminium alloy. Another test is performed on a fictitious material exhibiting a distribution of the anisotropy parameters that would lead to the occurrence of 8 ears in a cylindrical deep-drawing process.

KEYWORDS: plasticity, yield criterion, anisotropic sheet metals

1 INTRODUCTION

The yield criterion is an essential component of the mechanical models used for the simulation of sheet metal forming processes. Its capability to describe the anisotropic behaviour of the material has a significant influence on the quality of the results.

Many researchers have been concerned in the development of more accurate yield criteria, especially during the last decade [1]. The performances of such models are closely related to the flexibility of their mathematical formulation. In general, the flexibility is enhanced by including a larger number of material parameters in the yield criterion. As a consequence, the identification procedure needs more experimental data as input. The tests performed for obtaining this data have gradually extended their investigation area from uniaxial tension to biaxial tension, plane-strain tension and even pure shearing.

The yield criteria involving a large number of material parameters are also more complex from the mathematical point of view. This characteristic represents in many cases a serious drawback, especially when the computational efficiency is sought by the users.

A reasonable balance between the accuracy, computational efficiency, identification costs, and mathematical complexity is achieved by the models that use seven or eight experimental values as input. Such yield criteria have been proposed by Barlat et al. [2], Banabic et al. [3] and Cazacu [4].

When the plasticity of highly-anisotropic sheet metals must be described, the use of more complex models is unavoidable [5, 6, 7]. In these situations, the quality of the simulation results cannot be ensured without having an accurate description of the yield surface [8].

In order to enhance the flexibility of the BBC2005 yield criterion implemented in AutoForm 4.1 [9], the authors propose a new version of this model (BBC2008) expressed as a finite series that can be expanded to retain more or less terms, depending on the volume of experimental data. Two identification strategies (using 8 and 16 input values) will be described in the next sections, together with a series of tests aiming to prove the capabilities of the new yield criterion.

2 CONSTITUTIVE ASSUMPTIONS

The sheet metal is assumed to behave as a plastically orthotropic membrane under plane-stress conditions. We use the following description of the yield surface [10]:

$$\bar{\sigma}(\sigma_{\alpha\beta}) - Y = 0 \tag{1}$$

where $\bar{\sigma}(\sigma_{\alpha\beta}) \geq 0$ is the equivalent stress defined in §3, $Y > 0$ is the yield parameter, and $\sigma_{\alpha\beta} = \sigma_{\beta\alpha}$

* 15 C. Daicoviciu, 400020 Cluj-Napoca, Romania
Phone: +0040 264 401733, Fax: +0040 264 415603, email address: banabic@tcm.utcluj.ro

$(\alpha, \beta = 1, 2)$ are planar components of the stress tensor expressed in an orthonormal basis superimposed to the axes of plastic orthotropy: 1 – rolling direction (RD), 2 – transverse direction (TD), 3 – normal direction (ND). The other components are subjected to the constraint

$$\sigma_{3i} = \sigma_{i3} = 0, \quad i = 1, 2, 3 \tag{2}$$

arising from the plane-stress hypothesis. Whenever not specified, the following convention will be adopted: Latin subscripts take the values 1, 2 and 3, while the Greek ones take only the values 1 and 2.
The equivalent stress defined in §3 does not enforce constraints on the choice of the parameter Y. In fact, any quantity representing a yield stress can act as Y. For example, Y may be the uniaxial yield stress Y_θ associated to a planar direction defined by the angle θ measured from RD, an average of several uniaxial yield stresses, or the biaxial yield stress corresponding to the tension along RD and TD.
The flow rule associated to the yield surface described by Eqn (1) is [10]

$$\dot{\varepsilon}^{(p)}_{\alpha\beta} = \dot{\lambda} \frac{\partial \bar{\sigma}}{\partial \sigma_{\alpha\beta}} \tag{3}$$

where $\dot{\varepsilon}^{(p)}_{\alpha\beta} = \dot{\varepsilon}^{(p)}_{\beta\alpha}$ are planar components of the plastic strain-rate tensor (expressed in the same basis as the corresponding components of the stress tensor), and $\dot{\lambda} \geq 0$ is a scalar multiplier (its significance is not essential for our discussion). The out of plane components of the plastic strain-rate are subjected to the constraints

$$\dot{\varepsilon}^{(p)}_{3\alpha} = \dot{\varepsilon}^{(p)}_{\alpha 3} = 0, \quad \dot{\varepsilon}^{(p)}_{33} = -\dot{\varepsilon}^{(p)}_{11} - \dot{\varepsilon}^{(p)}_{22} \tag{4}$$

arising from the plane-stress hypothesis and the isochoric character of the plastic deformation [10].

3 EQUIVALENT STRESS

The equivalent stress used in Eqn (1) is defined as follows:

$$\begin{aligned}
&\frac{\bar{\sigma}^{2k}}{w-1} = \sum_{i=1}^{s} \Big\{ w^{i-1} \Big\{ \big[L^{(i)} + M^{(i)} \big]^{2k} + \big[L^{(i)} - M^{(i)} \big]^{2k} \Big\} + \\
&\qquad w^{s-i} \Big\{ \big[M^{(i)} + N^{(i)} \big]^{2k} + \big[M^{(i)} - N^{(i)} \big]^{2k} \Big\} \Big\} \\
&k, s \in \mathbf{N}^* \\
&w = (3/2)^{1/s} > 1 \\
&L^{(i)} = \ell^{(i)}_1 \sigma_{11} + \ell^{(i)}_2 \sigma_{22} \\
&M^{(i)} = \sqrt{\big[m^{(i)}_1 \sigma_{11} - m^{(i)}_2 \sigma_{22} \big]^2 + \big[m^{(i)}_3 (\sigma_{12} + \sigma_{21}) \big]^2} \\
&N^{(i)} = \sqrt{\big[n^{(i)}_1 \sigma_{11} - n^{(i)}_2 \sigma_{22} \big]^2 + \big[n^{(i)}_3 (\sigma_{12} + \sigma_{21}) \big]^2} \\
&\ell^{(i)}_1, \ell^{(i)}_2, m^{(i)}_1, m^{(i)}_2, m^{(i)}_3, n^{(i)}_1, n^{(i)}_2, n^{(i)}_3 \in \mathbf{R}.
\end{aligned} \tag{5}$$

The quantities denoted k, $\ell^{(i)}_1, \ell^{(i)}_2, m^{(i)}_1, m^{(i)}_2, m^{(i)}_3, n^{(i)}_1, n^{(i)}_2, n^{(i)}_3$ $(i = 1, \ldots, s)$ are material parameters. One may prove that $k \in \mathbf{N}^*$ is a sufficient condition for the convexity of the yield surface defined by Eqns (1) and (5). From this point of view, there is no constraint acting on the admissible values of the other material parameters.
It is easily noticeable that Eqn (5) reduces to the isotropic formulation proposed by Barlat and Richmond [11] if

$$\begin{aligned}
&\ell^{(i)}_1 = \ell^{(i)}_2 = m^{(i)}_1 = m^{(i)}_2 = m^{(i)}_3 = n^{(i)}_1 = n^{(i)}_2 = n^{(i)}_3 = 1/2, \\
&i = 1, \ldots, s
\end{aligned} \tag{6}$$

Under these circumstances, the exponent k may be chosen as in Barlat and Richmond's model, i.e. according to the crystallographic structure of the sheet metal: $k = 3$ for BCC materials $(2k = 6)$, and $k = 4$ for FCC materials $(2k = 8)$.
The other parameters involved in Eqn (5) result from an identification procedure (see §4). Their number (n_p) is defined by the summation limit s:

$$n_p = 8s \tag{7}$$

Let n_e be the number of experimental values describing the plastic anisotropy. The summation limit should be chosen according to the following constraint:

$$n_p = 8s \leq n_e \tag{8}$$

i.e.

$$s \leq n_e/8, \quad s \in \mathbf{N}^* \tag{9}$$

Apparently, Eqn (5) is usable only when $n_e \geq 8$. In fact, it also works with less experimental values. When such a situation occurs, the summation limit should be $s = 1$, and the $n_e < 8$ identification constraints arisen from experiments should be accompanied by at least $8 - n_e$ artificial conditions involving the material parameters. For example, if $n_e = 6$, we may enforce the equalities $m^{(1)}_1 = n^{(1)}_1$ and $m^{(1)}_2 = n^{(1)}_2$.

4 IDENTIFICATION PROCEDURE

Due to the expandable structure of the yield criterion, many identification strategies can be devised. We shall restrict our discussion to a procedure that uses only normalized yield stresses and r-coefficients obtained from uniaxial and biaxial tensile tests.
Let Y_θ be the yield stress predicted by the yield criterion in the case of a uniaxial traction along the direction defined by the angle θ measured from RD. The planar components of the stress tensor are in this case

$$\sigma_{11}\big|_{\theta} = Y_{\theta}\cos^2\theta,\quad \sigma_{22}\big|_{\theta} = Y_{\theta}\sin^2\theta,\\ \sigma_{12}\big|_{\theta} = \sigma_{21}\big|_{\theta} = Y_{\theta}\sin\theta\cos\theta \tag{10}$$

After replacing them in Eqn (5), we get the associated equivalent stress

$$\bar{\sigma}\big|_{\theta} = Y_{\theta}F_{\theta} \tag{11}$$

where F_θ is defined by the relationships

$$\frac{F_{\theta}^{2k}}{w-1} = \sum_{i=1}^{s}\Big\{ w^{i-1}\Big\{\big[L_{\theta}^{(i)}+M_{\theta}^{(i)}\big]^{2k}+\big[L_{\theta}^{(i)}-M_{\theta}^{(i)}\big]^{2k}\Big\}+ w^{s-i}\Big\{\big[M_{\theta}^{(i)}+N_{\theta}^{(i)}\big]^{2k}+\big[M_{\theta}^{(i)}-N_{\theta}^{(i)}\big]^{2k}\Big\}\Big\}$$

$$L_{\theta}^{(i)} = \ell_1^{(i)}\cos^2\theta+\ell_2^{(i)}\sin^2\theta \tag{12}$$

$$M_{\theta}^{(i)} = \sqrt{\big[m_1^{(i)}\cos^2\theta - m_2^{(i)}\sin^2\theta\big]^2+\big[m_3^{(i)}\sin 2\theta\big]^2}$$

$$N_{\theta}^{(i)} = \sqrt{\big[n_1^{(i)}\cos^2\theta - n_2^{(i)}\sin^2\theta\big]^2+\big[n_3^{(i)}\sin 2\theta\big]^2}$$

Eqns (1) and (11) lead to the following expression of the normalized uniaxial yield stress:

$$y_{\theta} = \frac{Y_{\theta}}{Y} = \frac{1}{F_{\theta}} \tag{13}$$

The r-coefficient corresponding to the uniaxial traction along a direction inclined at the angle θ measured from RD is defined by the formula [10]

$$r_{\theta} = \frac{\dot{\varepsilon}_{\theta+90^{\circ}}^{(p)}}{\dot{\varepsilon}_{ND}^{(p)}} \tag{14}$$

where $\dot{\varepsilon}_{\theta+90^{\circ}}^{(p)}$ is the plastic strain-rate component associated to the $\theta+90^{\circ}$ planar direction, and $\dot{\varepsilon}_{ND}^{(p)}$ is the through-thickness component of the same tensor. After some simple mathematical manipulations, Eqn (14) becomes

$$r_{\theta} = \frac{F_{\theta}}{G_{\theta}} - 1 \tag{15}$$

where G_θ is defined by the relationships

$$\frac{F_{\theta}^{2k-1}G_{\theta}}{w-1} = \sum_{i=1}^{s}\Big\{ w^{i-1}\big[\hat{L}_{\theta}^{(i)}+\hat{M}_{\theta}^{(i)}\big]\big[L_{\theta}^{(i)}+M_{\theta}^{(i)}\big]^{2k-1}+ w^{i-1}\big[\hat{L}_{\theta}^{(i)}-\hat{M}_{\theta}^{(i)}\big]\big[L_{\theta}^{(i)}-M_{\theta}^{(i)}\big]^{2k-1}+ w^{s-i}\big[\hat{M}_{\theta}^{(i)}+\hat{N}_{\theta}^{(i)}\big]\big[M_{\theta}^{(i)}+N_{\theta}^{(i)}\big]^{2k-1}+ w^{s-i}\big[\hat{M}_{\theta}^{(i)}-\hat{N}_{\theta}^{(i)}\big]\big[M_{\theta}^{(i)}-N_{\theta}^{(i)}\big]^{2k-1}\Big\} \tag{16}$$

$$\hat{L}_{\theta}^{(i)} = \ell_1^{(i)}+\ell_2^{(i)}$$

$$\hat{M}_{\theta}^{(i)} = \big[m_1^{(i)}-m_2^{(i)}\big]\big[m_1^{(i)}\cos^2\theta - m_2^{(i)}\sin^2\theta\big]\big/M_{\theta}^{(i)}$$

$$\hat{N}_{\theta}^{(i)} = \big[n_1^{(i)}-n_2^{(i)}\big]\big[n_1^{(i)}\cos^2\theta - n_2^{(i)}\sin^2\theta\big]\big/N_{\theta}^{(i)}$$

together with Eqn (12).

Let us denote by Y_b the yield stress predicted in the case of a biaxial traction along RD and TD. The corresponding planar components of the stress tensor are

$$\sigma_{11}\big|_b = Y_b,\quad \sigma_{22}\big|_b = Y_b,\quad \sigma_{12}\big|_b = \sigma_{21}\big|_b = 0 \tag{17}$$

After replacing them in Eqn (5), we get the associated equivalent stress

$$\bar{\sigma}\big|_b = Y_b F_b \tag{18}$$

where F_b is defined by the relationships

$$\frac{F_b^{2k}}{w-1} = \sum_{i=1}^{s}\Big\{ w^{i-1}\Big\{\big[L_b^{(i)}+M_b^{(i)}\big]^{2k}+\big[L_b^{(i)}-M_b^{(i)}\big]^{2k}\Big\}+ w^{s-i}\Big\{\big[M_b^{(i)}+N_b^{(i)}\big]^{2k}+\big[M_b^{(i)}-N_b^{(i)}\big]^{2k}\Big\}\Big\} \tag{19}$$

$$L_b^{(i)} = \ell_1^{(i)}+\ell_2^{(i)},\ M_b^{(i)} = m_1^{(i)}-m_2^{(i)},\ N_b^{(i)} = n_1^{(i)}-n_2^{(i)}$$

Eqns (1) and (18) lead to the following expression of the normalized biaxial yield stress:

$$y_b = \frac{Y_b}{Y} = \frac{1}{F_b} \tag{20}$$

The r-coefficient corresponding to the biaxial traction along RD and TD is defined by the formula [2]

$$r_b = \frac{\dot{\varepsilon}_{TD}^{(p)}}{\dot{\varepsilon}_{RD}^{(p)}} \tag{21}$$

where $\dot{\varepsilon}_{RD}^{(p)}$ and $\dot{\varepsilon}_{TD}^{(p)}$ are the plastic strain-rate components associated to the rolling and transverse directions, respectively. After some simple mathematical manipulations, Eqn (21) becomes

$$r_b = \frac{F_b}{G_b} - 1 \tag{22}$$

where G_b is defined by the relationships

$$\frac{F_b^{2k-1}G_b}{w-1} = \sum_{i=1}^{s}\Big\{ w^{i-1}\big[\hat{L}_b^{(i)}+\hat{M}_b^{(i)}\big]\big[L_b^{(i)}+M_b^{(i)}\big]^{2k-1}+ w^{i-1}\big[\hat{L}_b^{(i)}-\hat{M}_b^{(i)}\big]\big[L_b^{(i)}-M_b^{(i)}\big]^{2k-1}+ w^{s-i}\big[\hat{M}_b^{(i)}+\hat{N}_b^{(i)}\big]\big[M_b^{(i)}+N_b^{(i)}\big]^{2k-1}+ w^{s-i}\big[\hat{M}_b^{(i)}-\hat{N}_b^{(i)}\big]\big[M_b^{(i)}-N_b^{(i)}\big]^{2k-1}\Big\} \tag{23}$$

$$\hat{L}_b^{(i)} = \ell_1^{(i)},\quad \hat{M}_b^{(i)} = m_1^{(i)},\quad \hat{N}_b^{(i)} = n_1^{(i)}$$

together with Eqn (19).

An identification procedure that strictly enforces a large number of experimental constraints on the yield criterion would be inefficient in practical applications. The failure probability of such a strategy increases when the external restrictions become stronger. Taking into account this aspect, the authors have developed an identification procedure based on the minimization of the following error-function:

$$E\left[\ell_1^{(i)},\ell_2^{(i)},m_1^{(i)},m_2^{(i)},m_3^{(i)},n_1^{(i)},n_2^{(i)},n_3^{(i)}\middle| i=1,\ldots,s\right]=$$

$$\sum_{\theta_j}\left[\frac{y_{\theta_j}^{(\exp)}}{y_{\theta_j}}-1\right]^2+\sum_{\theta_j}\left[r_{\theta_j}^{(\exp)}-r_{\theta_j}\right]^2+ \quad (24)$$

$$\left[\frac{y_b^{(\exp)}}{y_b}-1\right]^2+\left[r_b^{(\exp)}-r_b\right]^2$$

where θ_j represents an individual element from a finite set of angles defining the orientation of the specimens used in the uniaxial tensile tests. One may notice that Eqn (24) describes a square-distance between the experimental and predicted values of the anisotropy characteristics.

The minimization has been performed using the subroutine LMDIF1 included in the double-precision version of the MINPACK-1 library [12]. LMDIF1 implements a modified Levenberg-Marquardt algorithm. The most important advantage of this subroutine consists in the fact that the Jacobian of the error-function is evaluated numerically by forward-difference approximations.

5 PERFORMANCES OF THE YIELD CRITERION

The results of two tests are presented in this section of the paper. The first one makes reference to an AA2090-T3 aluminium alloy [8]. The second test is performed on a fictitious material (FM) exhibiting a distribution of the anisotropy characteristics that would lead to the occurrence of 8 ears in a cylindrical deep-drawing process [8]. Table 1 contains the mechanical parameters used as input data.

Two versions of the BBC2008 yield criterion have been evaluated from the point of view of their performances. They include 8 and 16 material coefficients, respectively, and correspond to the smallest values of the summation limit ($s=1$ and $s=2$). The identification of the BBC2008 (16 parameters) model has been performed using all the mechanical parameters listed in Table 1. In the case of BBC2008 (8 parameters), the input data has been restricted to the values $y_{0^\circ}^{(\exp)}$, $y_{45^\circ}^{(\exp)}$, $y_{90^\circ}^{(\exp)}$, $y_b^{(\exp)}$, $r_{0^\circ}^{(\exp)}$, $r_{45^\circ}^{(\exp)}$, $r_{90^\circ}^{(\exp)}$, and $r_b^{(\exp)}$. Tables 2 and 3 contain the results of the identification procedure. Figures 1 – 6 show a comparison between the experimental data and the predictions of the BBC2008 model referring to the planar distribution of the normalized yield stress and r-coefficient in uniaxial tension, as well as to the shape of the normalized yield surface.

The version with 8 parameters of the BBC2008 model is able to reproduce exactly all the input data used in the identification. This fact seems natural if

Table 1: *Anisotropy characteristics of the materials used for testing the yield criterion performances [8]*

Material	AA2090-T3	FM
Structure	FCC	FCC
$y_{0^\circ}^{(\exp)}$	1.0000	1.0000
$y_{15^\circ}^{(\exp)}$	0.9605	1.0200
$y_{30^\circ}^{(\exp)}$	0.9102	1.0450
$y_{45^\circ}^{(\exp)}$	0.8114	1.0500
$y_{60^\circ}^{(\exp)}$	0.8096	1.0450
$y_{75^\circ}^{(\exp)}$	0.8815	1.0200
$y_{90^\circ}^{(\exp)}$	0.9102	1.0000
$y_b^{(\exp)}$	1.0350	1.0000
$r_{0^\circ}^{(\exp)}$	0.2115	0.6000
$r_{15^\circ}^{(\exp)}$	0.3269	1.0000
$r_{30^\circ}^{(\exp)}$	0.6923	0.7500
$r_{45^\circ}^{(\exp)}$	1.5769	0.3000
$r_{60^\circ}^{(\exp)}$	1.0385	0.7500
$r_{75^\circ}^{(\exp)}$	0.5384	1.0000
$r_{90^\circ}^{(\exp)}$	0.6923	0.6000
$r_b^{(\exp)}$	0.6700	1.0000

Table 2: *Coefficients of the BBC2008 yield criterion with 16 parameters* $(s=2)$

Material	AA2090-T3	FM
k	4	4
w	1.224745	1.224745
$\ell_1^{(1)}$	0.130866	0.405620
$\ell_2^{(1)}$	0.621742	0.405620
$m_1^{(1)}$	0.783422	0.767194
$m_2^{(1)}$	0.660402	0.767194
$m_3^{(1)}$	0.000079	0.000192
$n_1^{(1)}$	0.110991	0.001363
$n_2^{(1)}$	0.048245	0.001368
$n_3^{(1)}$	0.307522	0.644695
$\ell_1^{(2)}$	1.033922	0.532837
$\ell_2^{(2)}$	-0.071963	0.532837
$m_1^{(2)}$	0.000113	0.274012
$m_2^{(2)}$	0.000077	0.274012
$m_3^{(2)}$	0.538047	0.553576
$n_1^{(2)}$	0.055764	0.597125
$n_2^{(2)}$	1.018603	0.597125
$n_3^{(2)}$	0.778150	0.381241

Table 3: *Coefficients of the BBC2008 yield criterion with 8 parameters* $(s = 1)$

Material	AA2090-T3	FM
k	4	4
w	1.500000	1.500000
$\ell_1^{(1)}$	0.449938	0.500000
$\ell_2^{(1)}$	0.513218	0.500000
$m_1^{(1)}$	0.630315	0.532391
$m_2^{(1)}$	0.601445	0.532391
$m_3^{(1)}$	0.727299	0.505797
$n_1^{(1)}$	0.153818	0.425618
$n_2^{(1)}$	0.479391	0.425618
$n_3^{(1)}$	0.499818	0.356739

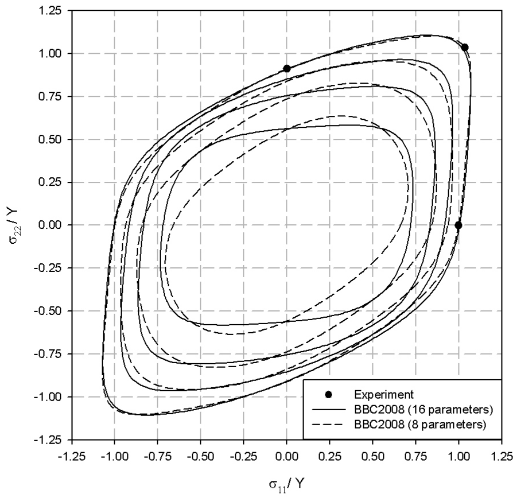

Figure 3: *Normalized yield surface predicted by the BBC2008 model for an AA2090-T3 aluminium alloy*

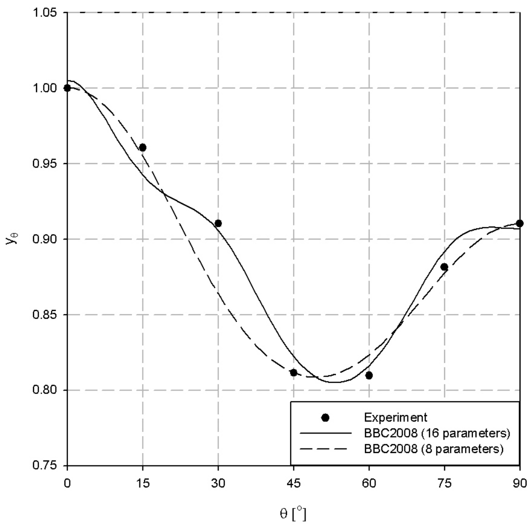

Figure 1: *Planar distribution of the uniaxial yield stress predicted by the BBC2008 model for an AA2090-T3 aluminium alloy*

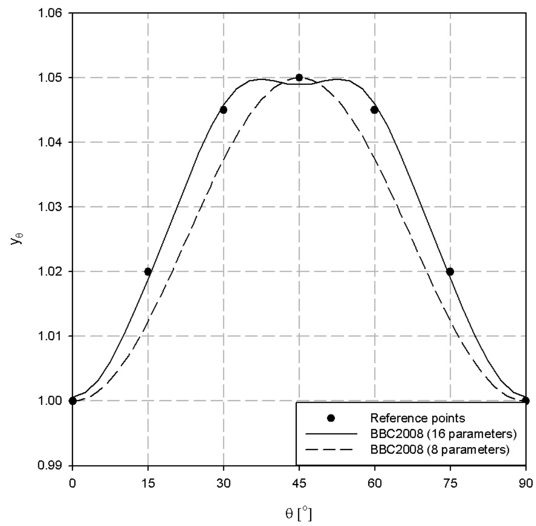

Figure 4: *Planar distribution of the uniaxial yield stress predicted by the BBC2008 model for the fictitious material FM*

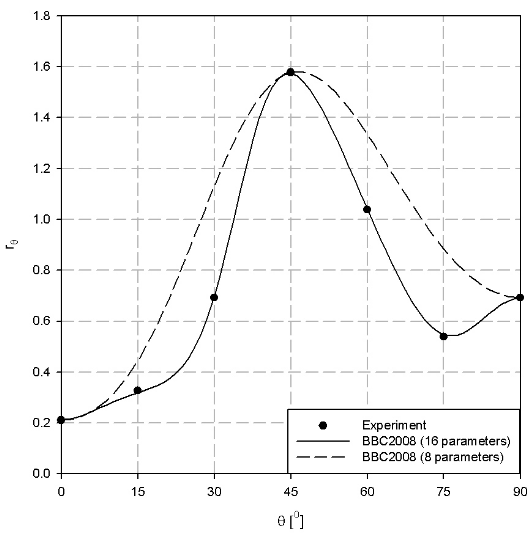

Figure 2: *Planar distribution of the r-coefficient predicted by the BBC2008 model for an AA2090-T3 aluminium alloy*

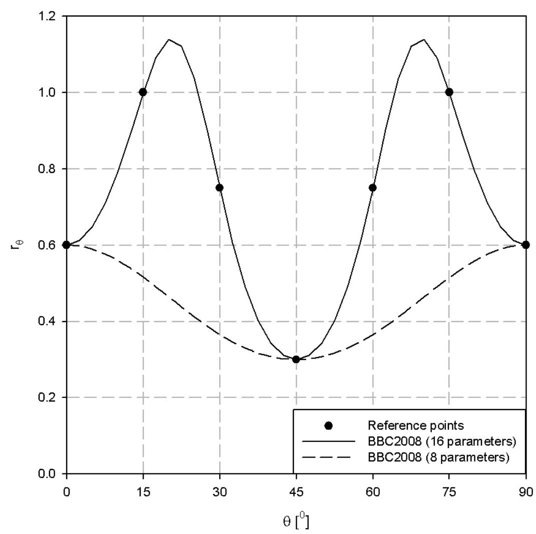

Figure 5: *Planar distribution of the r-coefficient predicted by the BBC2008 model for the fictitious material FM*

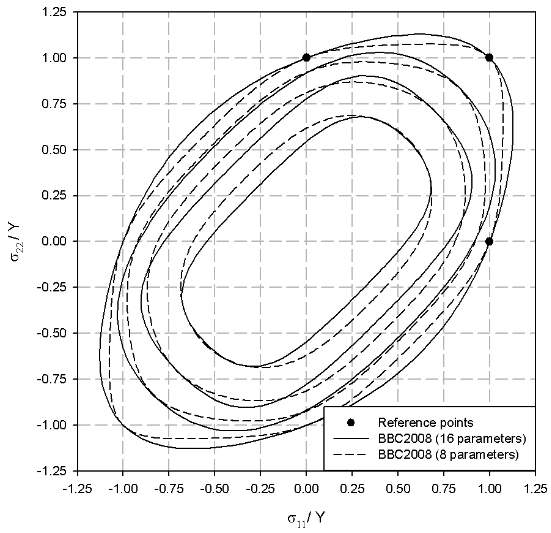

Figure 6: *Normalized yield surface predicted by the BBC2008 model for the fictitious material FM*

we take into account that the yield surface is subjected to less constraints in this case. Nevertheless, the predictions of the BBC2008 model with 16 parameters are superior to those given by the 8-parameter version both for the AA2090-T3 aluminium alloy and the fictitious material FM. The improvement is noticeable especially in the case of the r-coefficients. This capability of the 16-parameter version is relevant for the accurate prediction of the thickness when simulating sheet metal forming processes.

In the case of the fictitious material FM, the planar distribution of the r-coefficient (Figure 5) predicted by the BBC2008 yield criterion with 8 parameters is very inaccurate. This model would not be able to predict the occurrence of more than 4 ears at the top edge of a cup deep-drawn from a circular blank. In contrast, the variation of the r-coefficient described by BBC2008 with 16 parameters closely follows the reference data. According to Figure 5, this model would predict the occurrence of 8 ears as reported by Yoon et al. [8].

6 CONCLUSIONS

The authors have proposed a plane-stress yield criterion in the form of a finite series that can be expanded to retain more or less terms, depending on the volume of experimental data. As compared with other formulations described in the literature, the new model does not use linear transformations of the stress tensor. Due to this fact, its computational efficiency should be superior in the simulation of sheet metal forming processes. Even in the most reduced form, the yield criterion is able to give a sufficiently accurate description of medium or high anisotropies. When a larger number of terms are retained, the model can be used to capture the occurrence of more than 4 ears in a cylindrical deep-drawing process.

7 REFERENCES

[1] Banabic D., Barlat F., Cazacu O., Kuwabara T.: *Anisotropy and formability*, In: *Advances in Material Forming-ESAFORM 10 Years on*, (Editors: Chinesta F., Cueto, E.). Springer-Verlag Berlin Heidelberg New York, 143-173, 2007.

[2] Barlat F., Brem J.C., Yoon J.W., Chung K., Dick R.E., Lege D.J., Pourboghrat F., Choi S.H., Chu E.: *Plane stress yield function for aluminium alloy sheets. Part I: Theory.* International Journal of Plasticity, 19(12):1297-1319, 2003.

[3] Banabic D., Aretz H., Comsa D.S., Paraianu L.: *An improved analytical description of orthotropy in metallic sheets.* International Journal of Plasticity, 21(3):493-512, 2005.

[4] Cazacu O., Barlat F.: *Application of representation theory to describe yielding of anisotropic aluminum alloys.* International Journal of Engineering Science, 41(12):1367-1385, 2003.

[5] Barlat F., Aretz H., Yoon J.W., Karabin M.E., Brem J.C., Dick R.E.: *Linear transformation-based anisotropic yield functions.* International Journal of Plasticity, 21(5):1009–1039, 2005.

[6] Vegter H., van den Boogaard A.H.: *A plane stress yield function for anisotropic sheet material by interpolation of biaxial stress states*, International Journal of Plasticity, 22(3):557-580, 2006.

[7] Soare S., Yoon J.W., Cazacu O.: *On the use of homogeneous polynomials to develop anisotropic yield functions with applications to sheet forming.* International Journal of Plasticity, doi:10.1016/j.ijplas.2007.07.016, 2007.

[8] Yoon J.W., Barlat F., Dick R.E., Karabin M.E.: *Prediction of six or eight ears in a drawn cup based on new anisotropic yield function.* International Journal of Plasticity, 22(1):174-193, 2006.

[9] Banabic D., Comsa D.S., Sester M., Selig M., Kubli W., Mattiasson K., Sigvant M.: *Influence of constitutive equations on the accuracy of prediction in sheet metal forming simulation* , In: 7th International Conference NUMISHEET 2008.

[10] Hill, R.: *The Mathematical Theory of Plasticity*. Oxford University Press, 1998.

[11] Barlat F., Richmond O.: *Prediction of tricomponent plane stress yield surfaces and associated flow and failure behavior of strongly textured F.C.C. polycristalline sheets.* Materials Science and Engineering, 95(1):15-29, 1987.

[12] http://www.netlib.org/minpack/index.html