Available online at www.sciencedirect.com
SCIENCE DIRECT®
International Journal of Plasticity 21 (2005) 493–512
INTERNATIONAL JOURNAL OF Plasticity
www.elsevier.com/locate/ijplas

An improved analytical description of orthotropy in metallic sheets

D. Banabic [a,c,*], H. Aretz [b], D.S. Comsa [a], L. Paraianu [a]

[a] *Department of Mechanical Engineering, Technical University of Cluj-Napoca, Bd. Muncii nr. 103–105, 3400 Cluj-Napoca, Romania*
[b] *Institute of Metal Forming (IBF), RWTH Aachen University, Intzestrasse 10, 52056 Aachen, Germany*
[c] *Institute for Metal Forming Technology (IFU), Stuttgart University, Holzgartenstrasse 17, 70174 Stuttgart, Germany*

Received in final revised form 15 February 2004
Available online 6 May 2004

Abstract

The correct description of initial plastic anisotropy of metallic sheets plays a key role in modelling of sheet forming processes since prediction of material flow, residual stresses and springback as well as wrinkling and limiting strains are significantly affected by the phenomenological yield function applied in the analysis. In the last decades considerable improvement of anisotropic yield criteria has been achieved. Among these, the yield criterion proposed by Paraianu et al. [An improvement of the BBC2000 yield criterion. In: Proceedings of the ESAFORM 2003 Conference] is one of most promising plane stress yield criteria available for orthotropic sheet materials. This work aims to improve this yield criterion with respect to flexibility. The capabilities of the modified yield function will be demonstrated by applications to an anisotropic aluminium alloy sheet material.

Keywords: Sheet metal forming; Plastic anisotropy; Yield function; Plane stress

* Corresponding author. Tel.: +49-711-1214197; fax: +49-711-1213839.
E-mail address: banabic@ifu.uni-stuttgart.de (D. Banabic).

0749-6419/$ - see front matter
doi:10.1016/j.ijplas.2004.04.003

1. Introduction

For computer simulation of sheet metal forming processes a quantitative description of plastic anisotropy is required. Nowadays, there are several possibilities to account for plastic anisotropy which is mainly caused by crystallographic texture and microstructure of the considered material and its evolution during the forming process. Macroscopic plasticity theory is most frequently used due to its simplicity and its relatively low computational effort in numerical analysis. For the case of an isotropic metallic material, the well-known von Mises yield function is often sufficient to describe yielding. This is, however, not true for anisotropic materials, especially sheet metals. In order to take into account anisotropy, von Mises' yield function can be modified by introducing additional parameters. These parameters may be adjusted to a set of experimental data obtained by subjecting the considered material to mechanical tests. Hill's quadratic yield function (Hill, 1948; Hill, 1998) is the most frequently used yield function of this type. The reason lies in the fact that this yield function is very easy to handle in analytical and numerical calculations. One of the major drawbacks of Hill's criterion is its inability to describe the so-called anomalous behavior often observed in aluminium alloy sheets.

Thereafter, several scientists have proposed more and more sophisticated yield functions for anisotropic materials. Hill (1979, 1990) himself improved his criterion. However, it was stated by Hill (1993) that none of existing formulations is able to represent the behavior of a material exhibiting a tensile yield stress almost equal in value along the rolling and transverse direction, while r-values vary strongly with the angle to the rolling direction. To overcome this problem he proposed a new criterion (Hill, 1993).

Another important research direction in the field of yield function development was initiated by Hosford (1972), who, based on the results of polycrystal calculations, introduced a non-quadratic isotropic yield function. Hosford's criterion was later extended by Barlat and collaborators to anisotropic materials including shear stresses (Barlat and Richmond, 1987; Barlat and Lian, 1989; Barlat et al., 1991). Barlat et al. (1991) have developed a six-component yield function which is an extension of Hosford's yield function to anisotropy. Anisotropy was introduced by means of a linear transformation of the stress tensor. Later, this function was extended by Barlat et al. (1997a,b). Karafillis and Boyce (1993) have generalized the idea of linear stress transformation and suggested a yield function consisting of the sum of two convex functions. Banabic et al. (2000a,b) and Paraianu et al. (2003) extended the plane stress yield function introduced by Barlat and Lian (1989). Barlat et al. (2003) extended the concept of linear stress transformation and introduced two linear stress transformations.

During the last two decades, many other yield functions have been developed in order to improve agreement with the experimental results. For instance, Bassani (1977) has introduced a non-quadratic yield criterion; Gotoh (1977) introduced a fourth degree polynomial yield function; Budiansky (1984) proposed a yield function formulated as a parametric expression in polar coordinates which was extended by Tourki et al. (1994); Vegter et al. (1995) proposed a representation of the yield function using Bezier's interpolation and mechanical tests results directly; Cazacu and Barlat (2001) developed a yield function by introducing generalized invariants of the classical invariants of the stress deviator. More recently, Stoughton and Yoon (2004) used a non-associated flow rule in conjunction with Hill's, 1948 yield function. Bron and Besson (2004) extended the idea of Barlat et al. (2003) of using two linear stress transformations with respect to spacial stress state. More detailed reviews of numerous anisotropic yield criteria are given in the references (Banabic et al., 2000a; Barlat et al., 2002; Aretz, 2003).

The present article aims at improving the flexibility of the eight-parameters yield function introduced by Paraianu et al. (2003). Some applications to an anisotropic aluminium alloy sheet material confirm the capabilities of the new criterion.

2. Improvement of the BBC2002 yield criterion

2.1. BBC2002 yield criterion

In order to separate the elastic from the plastic state of deformation in a metallic workpiece subjected to forming, a yield function of the form

$$F(\boldsymbol{\sigma},\bar{\eta}) = \bar{\sigma}(\boldsymbol{\sigma}) - Y_{\text{ref}}(\bar{\eta}) \leqslant 0 \tag{1}$$

is commonly utilized in the phenomenological theory of plasticity. F denotes the yield function which depends on the Cauchy stress and the accumulated equivalent plastic strain $\bar{\eta}$ given by

$$\bar{\eta} = \int \dot{\bar{\eta}}\mathrm{d}t, \qquad \dot{\bar{\eta}} = \frac{\boldsymbol{\sigma} : \mathbf{d}^p}{\bar{\sigma}}, \tag{2}$$

where $\mathbf{d}^p$ is the plastic strain rate tensor. (Only isotropic hardening is considered in the present work, but the analysis made here may be easily extended to kinematic hardening.) $\bar{\sigma}$ denotes the equivalent stress. Y_{ref} is the instantaneous reference yield stress of the material. Any testing procedure (tensile, torsion, compression) may be used to obtain the flow curve $Y_{\text{ref}}(\bar{\eta})$. In order to account for plastic anisotropy, extra anisotropy parameters c_i, $i = 1, 2, 3, \ldots$, may be introduced and the anisotropic yield function may be written in the following generalized form:

$$F(c_i,\boldsymbol{\sigma},\bar{\eta}) = \bar{\sigma}(c_i,\boldsymbol{\sigma}) - Y_{\text{ref}}(\bar{\eta}) \leqslant 0 \tag{3}$$

with

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{4}$$

Based on the work of Barlat and Lian (1989) the BBC2002 yield criterion (Paraianu et al., 2003) was developed and is briefly presented here for the reader's convenience:

$$F(a, M, N, P, Q, R, S, T, k, \boldsymbol{\sigma}, \bar{\eta}) = \bar{\sigma}(a, M, N, P, Q, R, S, T, k, \boldsymbol{\sigma}) - Y_{\text{ref}}(\bar{\eta}) = 0. \tag{5}$$

Herein, $a, M, N, P, Q, R, S, T, k$ are anisotropy parameters as will be shown below. The equivalent stress $\bar{\sigma}$ is defined as

$$\bar{\sigma}(a,M,N,P,Q,R,S,T,k,\boldsymbol{\sigma}) = \left[a\cdot(\Gamma+\Psi)^{2k} + a\cdot(\Gamma-\Psi)^{2k} + (1-a)\cdot(2\Lambda)^{2k}\right]^{1/2k}. \tag{6}$$

k is an integer exponent. For bcc materials, the choice $k = 3$ yields satisfactory agreement of the yield locus shape with a yield locus calculated by means of a polycrystal Taylor model. For fcc materials, the same is true for $k = 4$. The terms Γ, Ψ and Λ are given as follows:

$$\begin{aligned}
\Gamma &= M\sigma_{11} + N\sigma_{22},\\
\Psi &= \sqrt{(P\sigma_{11} - Q\sigma_{22})^2 + R^2\sigma_{12}\sigma_{21}},\\
\Lambda &= \sqrt{(P\sigma_{11} - S\sigma_{22})^2 + T^2\sigma_{12}\sigma_{21}}.
\end{aligned} \tag{7}$$

Note that these equations are related to the principal stresses of a plane stress state. The anisotropy parameters serve as stress weighting parameters. The yield function is convex for $k \in \mathbb{N}^{\geqslant 1}$ and $0 \leqslant a \leqslant 1$. von Mises' yield criterion for isotropic materials is included as a special case.

The BBC2002 yield criterion behaves excellently in fitting of experimental data if a Newton solver is utilized to compute the anisotropy parameters for a given exponent k (see below). An alternative procedure to calculate the anisotropy parameters is to formulate an error function and to minimize this error (Banabic et al., 2000b). It may, however, be observed that the BBC2002 criterion behaves 'too stiffly' if it is calibrated by means of this method. An example is given in the appendix of the present paper. Thus, one may speculate that flexibility of the function is not always ensured and that it may fail in reproducing given experimental data. For this reason, an improved form of the BBC2002 criterion is proposed in the subsequent section.

2.2. Formulation of a new yield criterion

In order to overcome the shortcomings of the BBC2002 criterion explained above, the following modification of the BBC2002 yield criterion is proposed:

$$F(a, M, N, P, Q, R, S, T, k, \boldsymbol{\sigma}, \bar{\eta}) = \bar{\sigma}(a, M, N, P, Q, R, S, T, k, \boldsymbol{\sigma}) - Y_{\text{ref}}(\bar{\eta}) = 0 \tag{8}$$

with

$$\bar{\sigma}(a,M,N,P,Q,R,S,T,k,\boldsymbol{\sigma}) = \left[a\cdot(\Gamma+\Psi)^{2k} + a\cdot(\Gamma-\Psi)^{2k} + (1-a)\cdot(2\Lambda)^{2k}\right]^{1/2k}. \tag{9}$$

The terms Γ, Ψ and Λ are defined as follows:

$$\Gamma = \frac{\sigma_{11} + M\sigma_{22}}{2},$$
$$\Psi = \sqrt{\frac{(N\sigma_{11} - P\sigma_{22})^2}{4} + Q^2\sigma_{12}\sigma_{21}}, \tag{10}$$
$$\Lambda = \sqrt{\frac{(R\sigma_{11} - S\sigma_{22})^2}{4} + T^2\sigma_{12}\sigma_{21}}.$$

The yield function is convex for $k \in I\mathcal{N}^{\geqslant 1}$ and $0 \leqslant a \leqslant 1$. Due to the re-arranging of the parameters M, N, P, Q, R, S, T the yield function shows improved flexibility in the error minimization calibration method. Thus, the new formulation offers more mathematical possibilities for adjusting the anisotropy parameters to experimental data. In the following this modified yield criterion will be referred to as 'BBC2003'. An example for the improved flexibility of BBC2003 compared to BBC2002 is given in the appendix of the present article. The associated flow rule in terms of the BBC2003 yield function is also presented in the appendix of the present article.

It should be mentioned that a further increase in flexibility could easily be introduced in the new formulation if the Γ-term is written as

$$\Gamma = \frac{L\sigma_{11} + M\sigma_{22}}{2} \tag{11}$$

using an additional anisotropy parameter L. This increases the number of adjustable parameters from 8 to 9 (without counting the exponent k which is associated with the crystal structure and thus does not represent a variable in this context).

It should be mentioned that the yield function defined by Eqs. (8) and (9) can also be deduced from the recently published yield function 'Yld2000-2D' proposed by Barlat et al. (2003). Therefore, both yield functions, BBC2003 and Yld2000-2D, are not fundamentally different. Only their form differs slightly.

2.3. Required material data for sheet metal forming analysis

The minimum experimental data which should enter the yield function as input data includes:

- Three directional yield stresses obtained from uniaxial tensile tests along a direction at 0°, 45° and 90° to the rolling direction of the sheet. The associated yield stresses are denoted here as Y_0, Y_{45}, Y_{90}.
- Three r-values corresponding to 0°, 45°, 90° orientations (denoted here as r_0, r_{45}, r_{90}).
- The equibiaxial yield stress obtained by cross tensile test or bulge test (denoted here as Y_b).
- The equibiaxial r-value (ratio of plastic strain in transverse direction to plastic strain in rolling direction), denoted as r_b.

This makes a total of 8 experimental data points which should be used for the calibration of the yield function. BBC2002 as well as the yield function proposed in this paper (see Eqs. (9) and (10)) exhibits 8 coefficients which may be determined using the 8 experimental data points.

2.4. Determination of the anisotropy parameters

Eqs. (9) and (10) contain 8 coefficients (M, N, P, Q, R, S, T, a) while the exponent k is associated with the crystal structure of the sheet material and thus does not represent a variable in the following context. For the adjustment of the anisotropy parameters the following experimental data are considered to be given: Y_0, Y_{45}, Y_{90}, Y_b, r_0, r_{45}, r_{90}, r_b. From the tensor transformation rules the stress components in a tensile test specimen with orientation angle φ with respect to the original rolling direction are given as

$$\begin{aligned} \sigma_{11} &= Y_\varphi \cdot \cos^2 \varphi, \\ \sigma_{22} &= Y_\varphi \cdot \sin^2 \varphi, \\ \sigma_{12} &= \sigma_{21} = Y_\varphi \cdot \cos \varphi \cdot \sin \varphi. \end{aligned} \tag{12}$$

Y_φ is the yield stress of the tensile test specimen under uniaxial load. This gives three stress tensors $\boldsymbol{\sigma}_0$, $\boldsymbol{\sigma}_{45}$ and $\boldsymbol{\sigma}_{90}$, respectively, associated with the orientation angle $\varphi \in \{0°, 45°, 90°\}$.

For the equibiaxial tensile test (or, alternatively, the bulge test) the stress tensor components are given by $\sigma_{11} = \sigma_{22} = Y_b$, $\sigma_{12} = 0$. This results in a stress tensor denoted as $\boldsymbol{\sigma}_b$.

A directional r-value associated with the orientation angle φ can be calculated from

$$r_\varphi = -\left.\frac{\sin^2 \varphi \cdot \frac{\partial F}{\partial \sigma_{11}} - \sin 2\varphi \cdot \frac{\partial F}{\partial \sigma_{12}} + \cos^2 \varphi \cdot \frac{\partial F}{\partial \sigma_{22}}}{\frac{\partial F}{\partial \sigma_{11}} + \frac{\partial F}{\partial \sigma_{22}}}\right|_{\boldsymbol{\sigma}_\varphi} \tag{13}$$

with $\varphi \in \{0°, 45°, 90°\}$. The biaxial r-value, r_b, follows from

$$r_b = \left.\frac{\frac{\partial F}{\partial \sigma_{22}}}{\frac{\partial F}{\partial \sigma_{11}}}\right|_{\boldsymbol{\sigma}_b}. \tag{14}$$

As a result, one obtains the following set of 8 equations:

$$\left.\begin{aligned} &\bar{\sigma}(\boldsymbol{\sigma}_0, a, M, N, P, Q, R, S, T) - Y_{ref} = 0 \\ &\bar{\sigma}(\boldsymbol{\sigma}_{45}, a, M, N, P, Q, R, S, T) - Y_{ref} = 0 \\ &\bar{\sigma}(\boldsymbol{\sigma}_{90}, a, M, N, P, Q, R, S, T) - Y_{ref} = 0 \\ &\bar{\sigma}(\boldsymbol{\sigma}_b, a, M, N, P, Q, R, S, T) - Y_{ref} = 0 \\ &r_0(\boldsymbol{\sigma}_0, a, M, N, P, Q, R, S, T) - r_0^{exp} = 0 \\ &r_{45}(\boldsymbol{\sigma}_{45}, a, M, N, P, Q, R, S, T) - r_{45}^{exp} = 0 \\ &r_{90}(\boldsymbol{\sigma}_{90}, a, M, N, P, Q, R, S, T) - r_{90}^{exp} = 0 \\ &r_b(\boldsymbol{\sigma}_b, a, M, N, P, Q, R, S, T) - r_b^{exp} = 0 \end{aligned}\right\} \tag{15}$$

$r_{(\bullet)}^{\exp}$ denote the experimentally determined r-values while $r_{(\bullet)}$ are calculated according to the formulas given above. This equation system is non-linear with respect to the 8 unknown anisotropy parameters a, M, N, P, Q, R, S, T and may be solved by means of a Newton solver. As an alternative way to calculate the anisotropy parameters one may define an error function $\mathscr{E}$ by means of the Gaussian square of error (Aretz, 2003)

$$\mathscr{E}(a,M,N,P,Q,R,S,T) = \left(\frac{\bar{\sigma}_{\mathrm{b}} - Y_{\mathrm{ref}}}{Y_{\mathrm{ref}}}\right)^2 + \sum_{i=1}^{3}\left(\frac{\bar{\sigma}_{\varphi_i} - Y_{\mathrm{ref}}}{Y_{\mathrm{ref}}}\right)^2 + \left(\frac{r_{\mathrm{b}} - r_{\mathrm{b}}^{\exp}}{r_{\mathrm{b}}^{\exp}}\right)^2 + \sum_{i=1}^{3}\left(\frac{r_{\varphi_i} - r_{\varphi_i}^{\exp}}{r_{\varphi_i}^{\exp}}\right)^2 = \mathrm{Min.} \qquad (16)$$

with $\{\varphi_1, \varphi_2, \varphi_3\} = \{0°, 45°, 90°\}$ and to minimize this error function. Abbreviations such as

$$\bar{\sigma}_{\mathrm{b}} \equiv \bar{\sigma}(\boldsymbol{\sigma}_{\mathrm{b}}, a, M, N, P, Q, R, S, T)$$

have been utilized. There are different mathematical methods available to minimize such a function (Alt, 2002). One of the most convenient methods is the known method of steepest descent which is preferred here.

In the authors' opinion the error minimization procedure described above is an excellent engineering method to check a yield function's flexibility: only if it is possible to fit an anisotropic yield function using the error minimization procedure with sufficient quality to given experimental data the yield function is considered to be flexible enough for general purpose. BBC2000 (Banabic et al., 2000b) and BBC2002 (Paraianu et al., 2003) do not pass this flexibility test, although they can be fitted excellently to experimental data by means of a Newton solver. In contrast, the new proposal given in Eqs. (9)–(11) does pass this test and offers therefore a higher flexibility than the other 'BBC' approaches.

3. Applications

3.1. Experimental results

By varying the longitudinal and transverse force acting on a cruciform tensile specimen, any point of the yield locus in the range of biaxial tensile stress can be realized. Kreißig (1981) described a cross tensile specimen which was optimized by Müller (1996), see Fig. 1.

In order to determine the yield locus of a sheet material in the initial state without pre-straining, the nominal cross section can be used with good accuracy. The experimental yield locus presented in the present work has been determined in this way. The typical characterization of the yield locus is made using five experimental points

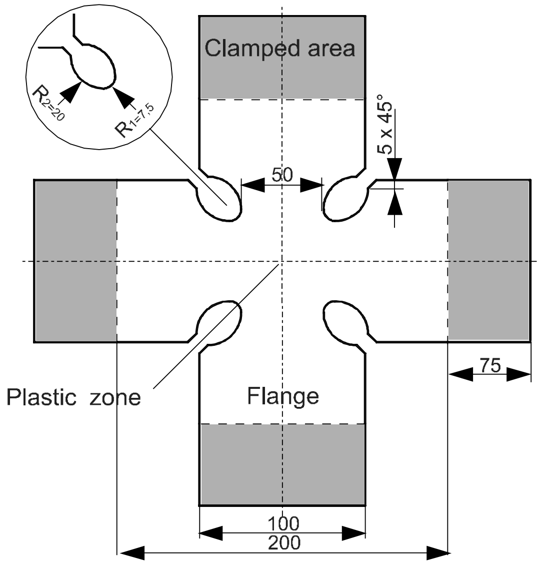

Fig. 1. Cruciform specimen for the biaxial tensile test (Müller, 1996). All dimensions are in millimeters.

under seven different ratios of the applied stresses: 1:0; 4:1; 2:1; 1:1; 1:2; 1:4; 0:1. Y_0, Y_{90} and Y_b are used for the adjustment of the yield function. The remaining points will be used to check the difference between experimental measurements and the yield function's prediction. All these points are located in the first quadrant of the yield locus (biaxial tension).

The biaxial tensile tests were carried out by means of a CNC biaxial tensile test device designed and built at the Institute of Metal Forming Technology, Stuttgart University, see Fig. 2.

The beginning of plastic yielding was monitored by temperature measurements according to the method of Sallat (1988). The temperature of the specimen was measured by an infrared thermo-couple positioned at an optimized distance from the specimen. During elastic straining, the specimen's temperature decreases by a fraction of a degree due to thermo-elastic cooling. When plastic flow begins, the temperature rises fast (Fig. 3).

In contrast to the standard definition of the yielding point the minimum of the temperature vs. elongation enables a definition without any arbitrariness. In general, this method overestimates the value of the yield stress compared to the classical one by using stress gauges.

The predictions of the BBC2003 yield criterion have been tested for an AA6181-T4 aluminum alloy sheet metal. Table 1 contains the most important mechanical parameters of the material under consideration. The initial thickness of the sheet was 1.13 mm. The measured yield stresses for various biaxial stress states are displayed in Table 2.

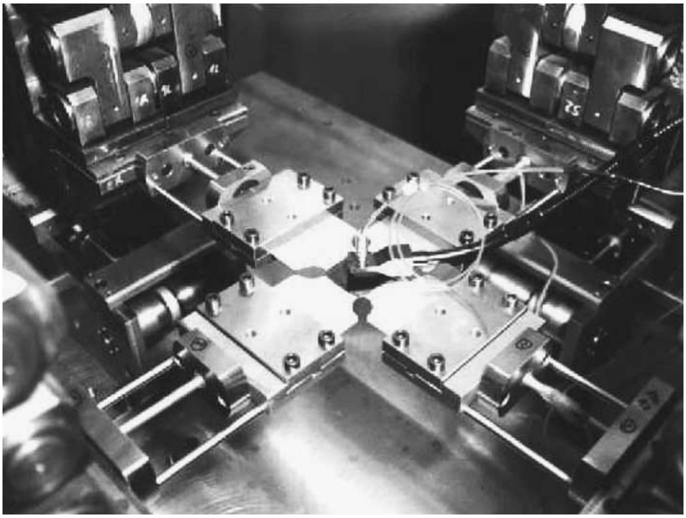

Fig. 2. Cruciform specimen and the temperature measurement device.

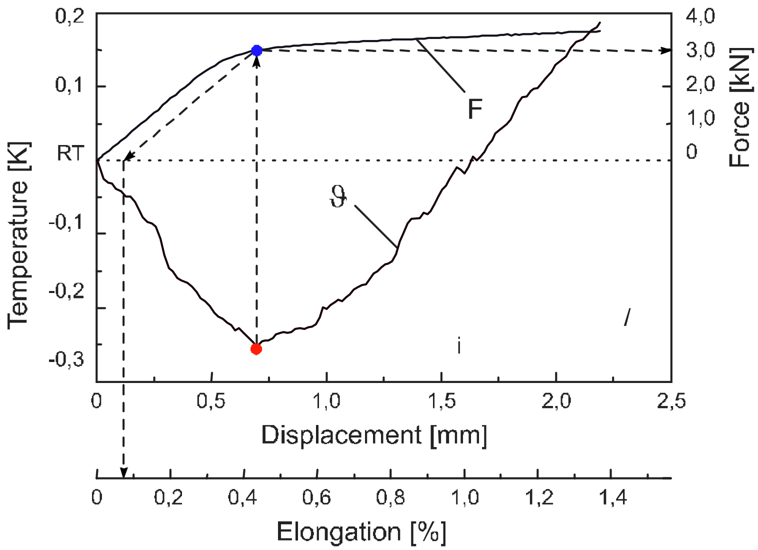

Fig. 3. Temperature vs. elongation obtained for standard tensile test specimen (Müller, 1996).

3.2. Comparison between theory and experiments

In the following the predictive capabilities of the BBC2003 yield criterion are demonstrated for the above mentioned AA6181-T4 aluminum sheet alloy. The error minimization identification and the Newton solver identification procedure are

Table 1
Selected mechanical parameters of the aluminum alloy AA6181-T4

Y_0 [MPa]	Y_{45} [MPa]	Y_{90} [MPa]	Y_b [MPa]	r_0 [–]	r_{45} [–]	r_{90} [–]	r_b [–]
142	138	137	134	0.672	0.606	0.821	0.820

Table 2
Measured yield stresses for various biaxial stress states of the aluminium alloy AA6181-T4

σ_{11} [MPa]	σ_{22} [MPa]	σ_{11} [MPa]	σ_{22} [MPa]
142	0	144.3	102.6
140	0	146.1	104.3
140.9	0	146.1	102.6
36.5	148.9	139.1	34.8
40	148.7	139.1	41.7
36.5	140.4	140.9	38.3
97.6	147.7	0	135
99.3	149.7	0	137
101.7	152.7	0	139
126.3	145.6		
125.9	139.1		
126.2	139.8		

Table 3
BBC2003 anisotropy parameters for the aluminum alloy AA6181-T4 calculated by means of the error function

a [–]	M [–]	N [–]	P [–]	Q [–]	R [–]	S [–]	T [–]	Y_{ref} [MPa]	k [–]
0.45985	1.12002	1.03533	0.986456	1.07315	0.95934	1.02048	0.98339	142	4

Table 4
BBC2003 anisotropy parameters for the aluminum alloy AA6181-T4 calculated by means of a Newton solver

a [–]	M [–]	N [–]	P [–]	Q [–]	R [–]	S [–]	T [–]	Y_{ref} [MPa]	k [–]
0.54340	1.09399	1.01843	0.97275	1.04557	0.98820	1.04013	1.00480	142	4

compared to each other by using the input data given in Table 1. Tables 3 and 4 show the values of the material parameters a, M, N, P, Q, R, S, T obtained by numerical identification by minimization of the error function and by means of a

Newton solver (Press, 1992), respectively. The corresponding values of k and Y_{ref} are also presented for completeness.

Figs. 4 and 5 show the predicted directional r-values and the uniaxial yield stresses, respectively, in comparison to experimental data. It may be seen that the Newton solver solution meets the experimental input data points exactly, while the calibration by means of the error minimization shows small discrepancies. According to the yield stresses the error minimization leads only to an (acceptable) deviation of the yield stress at 0° to the rolling direction while the remaining points are met exactly. As a consequence, the yield loci predicted by both identification methods are quite close to each other and agree with the experimental data very well (see Fig. 6). The same good agreement may be seen in the π-plane predictions shown in Fig. 7. The error minimization fitting resulted in the following predictions for the equibiaxial data: $Y_b = 135.12$ MPa, $r_b = 0.825$. On the other hand, the Newton solver fitting gave $Y_b = 134.00$ MPa, $r_b = 0.820$.

In general one may see that the Newton solver is better suited for parameters identification than the error minimization method. Nevertheless, the latter method is a very suitable engineering way to test a yield function's flexibility. With respect to this point, the new yield function BBC2003 passed this flexibility test. Perfect agreement with the experimental input data is, however, only achieved by utilizing a Newton solver identification.

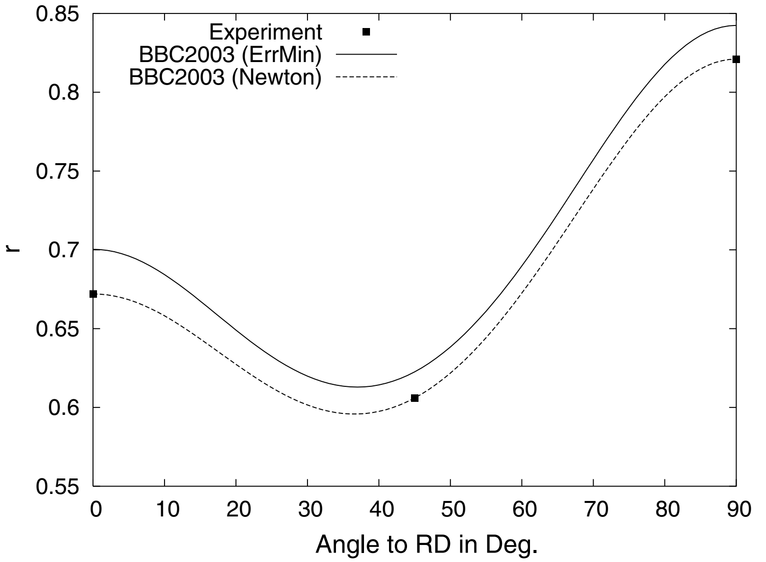

Fig. 4. Comparison of experimental and predicted directional r-values by parameters identification using the error minimization method and a Newton solver, respectively. 'RD' means 'rolling direction'.

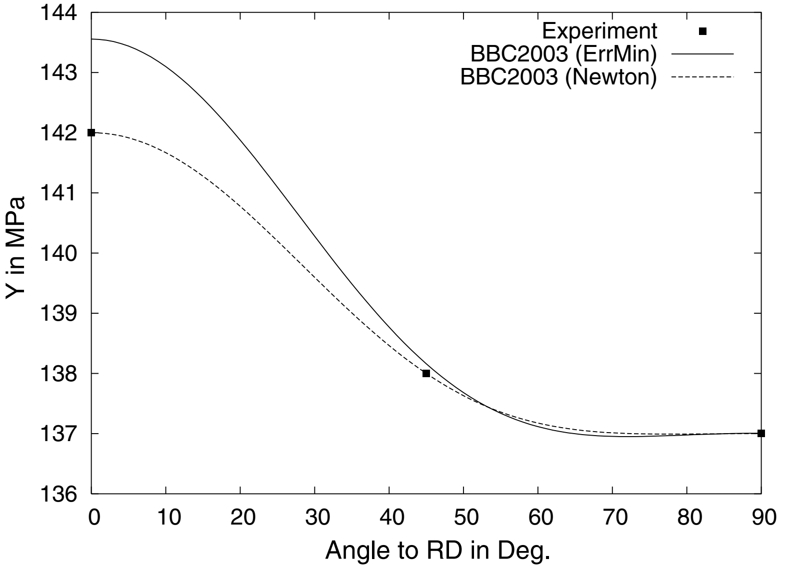

Fig. 5. Comparison of experimental and predicted directional yield stresses by parameters identification using the error minimization method and a Newton solver, respectively. 'RD' means 'rolling direction'.

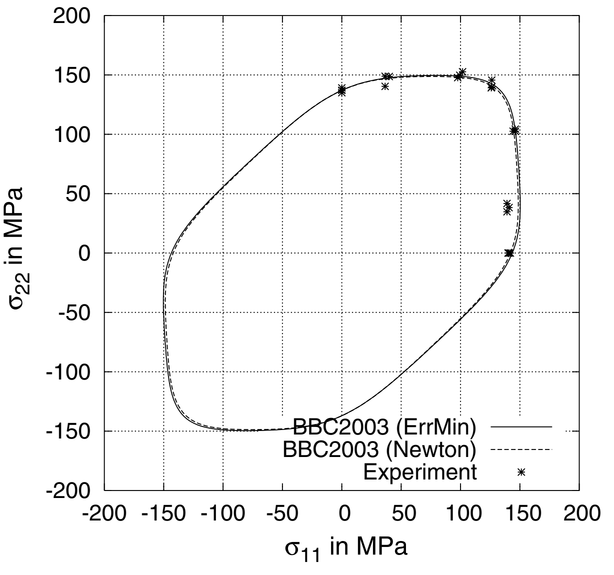

Fig. 6. Comparison of experimental and predicted yield loci by parameters identification using the error minimization method and a Newton solver, respectively.

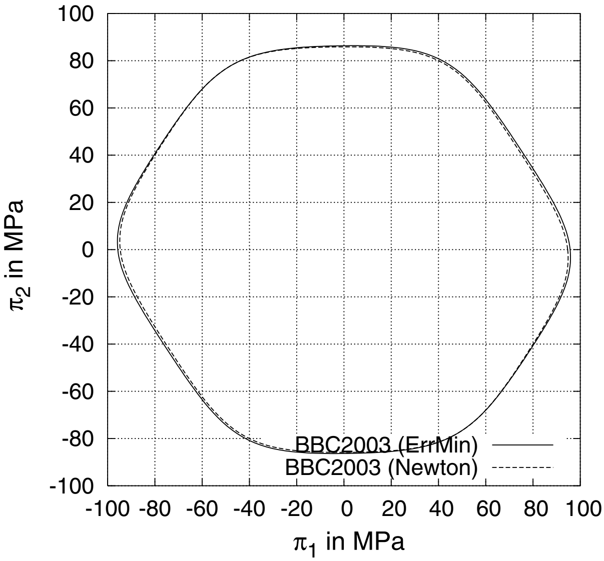

Fig. 7. Predicted yield loci in the π-plane by parameters identification using the error minimization method and a Newton solver, respectively.

4. Conclusions

A new yield criterion deduced from the one introduced by Paraianu et al. (2003) has been proposed. In comparison to the original proposed yield function the new one has an increased flexibility. The minimization of an error function as well as a Newton solver have been used for the numerical identification of the anisotropy coefficients for an AA6181-T4 aluminum alloy sheet. The first identification method (error function) lead to small discrepancies in prediction of directional yield stresses and *r*-values. By utilizing the second method for the parameters identification (Newton solver) all experimental input data points were met exactly by the new yield function. In both cases, the yield locus was in very good agreement. The capabilities and the simplicity of the proposed yield function make it very attractive for an implementation in finite element codes.

Acknowledgements

D. Banabic thanks Prof. Dr.-Ing. K. Siegert, Director of the Institute for Metal Forming Technology (IFU), Stuttgart University, for his useful advice during the stay at IFU.

H. Aretz gratefully acknowledges the financial support by the Deutsche Forschungsgemeinschaft within the Collaborative Research Centre SFB 370.

Special thanks to O.S. Hopperstad and to L. Neumann for a careful review of this manuscript.

Appendix A

A.1. Application of BBC2002 and BBC2003 to Al2090-T3

In the following the new yield function proposal BBC2003 will be compared with the BBC2002 criterion and the superior flexibility of BBC2003 will be demonstrated. For this purpose the material Al2090-T3 (Barlat et al., 2003) has been considered because it shows a very strong anisotropy and is frequently used as a testing material. The corresponding material data is given in Table 5.

In order to calibrate the yield functions BBC2002 and BBC2003 the following data has been selected as input data: Y_0/Y_0, Y_{45}/Y_0, Y_{90}/Y_0, Y_b/Y_0, r_0, r_{45}, r_{90}, r_b. (It should be mentioned that it does not matter if normalized yield stresses are utilized as input data.)

Figs. 8–11 display the results obtained by calibrating BBC2002 and BBC2003, respectively, by means of the error minimization method. The same code and the same numerical parameters have been utilized to calibrate the two yield functions. Additionally, the results obtained by fitting BBC2003 by means of a Newton solver are also shown in these figures. It may clearly be seen that the BBC2003 approach is significantly superior to BBC2002 especially with respect to the prediction of the equibiaxial yield stress as shown in Fig. 10. The results obtained by the Newton solver meet all experimental input data exactly and may be considered as a reference in the shown figures.

It should be mentioned that the superiority of the BBC2003 criterion compared to the BBC2002 approach has also been found for some other materials, but these results are not presented here.

A.2. Associated flow rule in terms of the BBC2003 yield function

In the phenomenological theory of plasticity the associated flow rule

Table 5
Mechanical parameters of the aluminum alloy Al2090-T3

Y_0/Y_0 [–]	Y_{15}/Y_0 [–]	Y_{30}/Y_0 [–]	Y_{45}/Y_0 [–]	Y_{60}/Y_0 [–]	Y_{75}/Y_0 [–]	Y_{90}/Y_0 [–]	Y_b/Y_0 [–]	Y_{ref}/Y_0 [–]
1.000	0.961	0.910	0.811	0.811	0.882	0.910	1.035	1.000
r_0 [–]	r_{15} [–]	r_{30} [–]	r_{45} [–]	r_{60} [–]	r_{75} [–]	r_{90} [–]	r_b [–]	k [–]
0.210	0.327	0.692	1.580	1.039	0.538	0.690	0.670	4

The yield stresses are normalized with respect to Y_0.

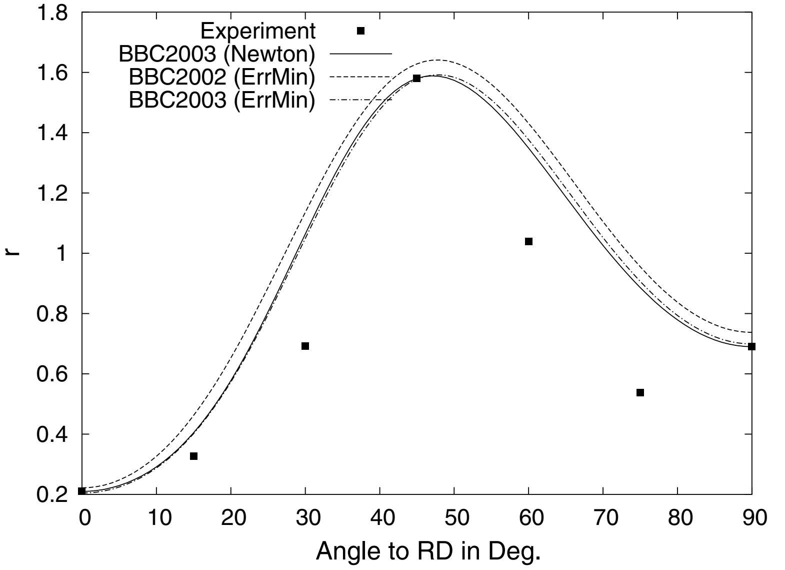

Fig. 8. Comparison of experimental and predicted directional *r*-values by parameters identification using the error minimization method and a Newton solver, respectively. 'RD' means 'rolling direction'.

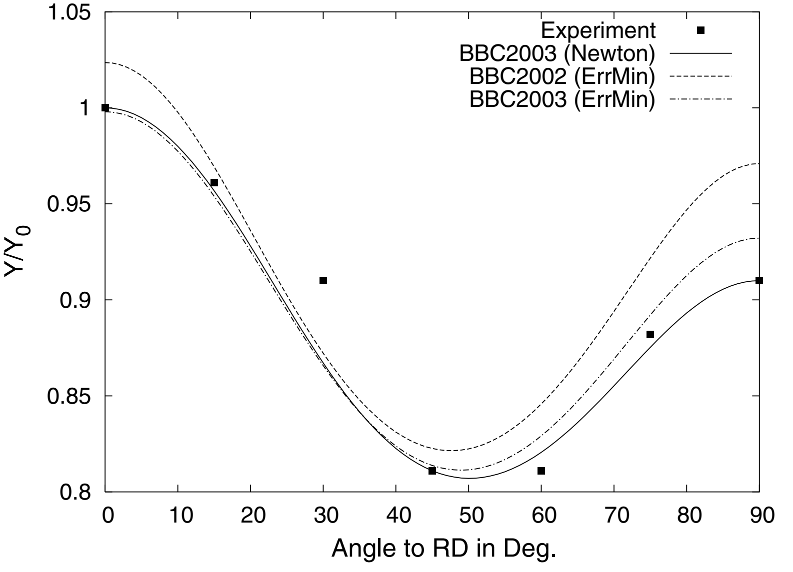

Fig. 9. Comparison of experimental and predicted directional yield stresses by parameters identification using the error minimization method and a Newton solver, respectively. 'RD' means 'rolling direction'.

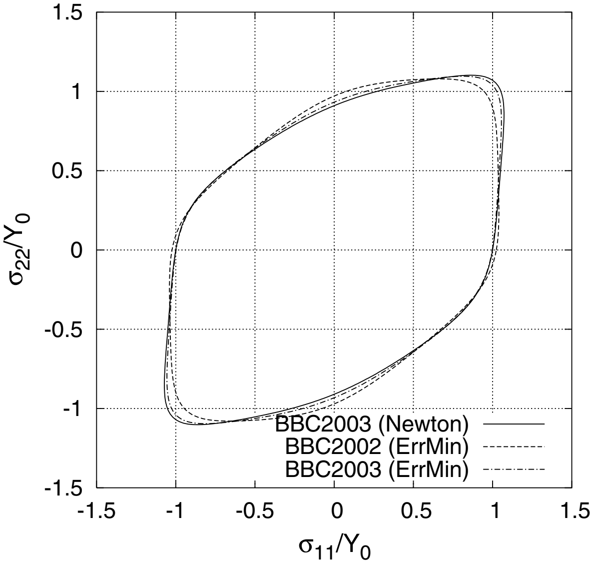

Fig. 10. Predicted yield loci by parameters identification using the error minimization method and a Newton solver, respectively.

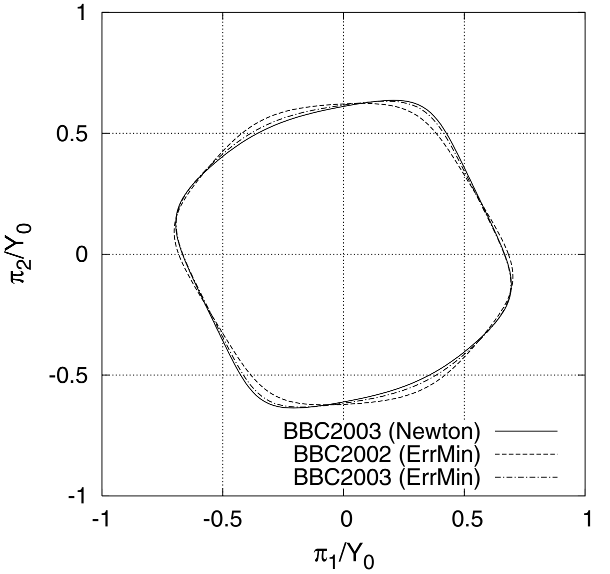

Fig. 11. Predicted yield loci in the π-plane by parameters identification using the error minimization method and a Newton solver, respectively.

$$\mathbf{d}^p = \dot{\lambda}\frac{\partial F}{\partial \boldsymbol{\sigma}} \tag{A.1}$$

gives the evolution of the plastic deformation. For the plane stress yield function BBC2003 given by Eqs. (9) and (10), only four non-zero gradient components can be calculated. They are given by

$$\left.\begin{aligned}
\frac{\partial F}{\partial \sigma_{11}} &= \frac{\partial F}{\partial \Gamma}\frac{\partial \Gamma}{\partial \sigma_{11}} + \frac{\partial F}{\partial \Psi}\frac{\partial \Psi}{\partial \sigma_{11}} + \frac{\partial F}{\partial \Lambda}\frac{\partial \Lambda}{\partial \sigma_{11}}\\
\frac{\partial F}{\partial \sigma_{22}} &= \frac{\partial F}{\partial \Gamma}\frac{\partial \Gamma}{\partial \sigma_{22}} + \frac{\partial F}{\partial \Psi}\frac{\partial \Psi}{\partial \sigma_{22}} + \frac{\partial F}{\partial \Lambda}\frac{\partial \Lambda}{\partial \sigma_{22}}\\
\frac{\partial F}{\partial \sigma_{12}} &= \frac{\partial F}{\partial \Gamma}\frac{\partial \Gamma}{\partial \sigma_{12}} + \frac{\partial F}{\partial \Psi}\frac{\partial \Psi}{\partial \sigma_{12}} + \frac{\partial F}{\partial \Lambda}\frac{\partial \Lambda}{\partial \sigma_{12}}\\
\frac{\partial F}{\partial \sigma_{21}} &= \frac{\partial F}{\partial \Gamma}\frac{\partial \Gamma}{\partial \sigma_{21}} + \frac{\partial F}{\partial \Psi}\frac{\partial \Psi}{\partial \sigma_{21}} + \frac{\partial F}{\partial \Lambda}\frac{\partial \Lambda}{\partial \sigma_{21}}
\end{aligned}\right\} \tag{A.2}$$

This equation contains some common partial derivatives and some specific ones. The common ones are given as follows:

$$\frac{\partial F}{\partial \Gamma} = A\cdot\left\{a\cdot(\Gamma+\Psi)^{2k-1} + a\cdot(\Gamma-\Psi)^{2k-1}\right\}, \tag{A.3}$$

$$\frac{\partial F}{\partial \Psi} = A\cdot\left\{a\cdot(\Gamma+\Psi)^{2k-1} - a\cdot(\Gamma-\Psi)^{2k-1}\right\}, \tag{A.4}$$

$$\frac{\partial F}{\partial \Lambda} = A\cdot\left\{2\cdot(1-a)\cdot(2\Lambda)^{2k-1}\right\}, \tag{A.5}$$

with

$$A \equiv \left[a\cdot(\Gamma+\Psi)^{2k} + a\cdot(\Gamma-\Psi)^{2k} + (1-a)\cdot(2\Lambda)^{2k}\right]^{(1/2k)-1} \tag{A.6}$$

The remaining specific partial derivatives are:

$$\frac{\partial \Gamma}{\partial \sigma_{11}} = \frac{1}{2}, \qquad \frac{\partial \Gamma}{\partial \sigma_{22}} = \frac{M}{2}, \qquad \frac{\partial \Gamma}{\partial \sigma_{12}} = 0, \tag{A.7}$$

$$\frac{\partial \Psi}{\partial \sigma_{11}} = \frac{\frac{1}{4}\cdot(N\sigma_{11} - P\sigma_{22})\cdot N}{\sqrt{\frac{1}{4}\cdot(N\sigma_{11} - P\sigma_{22})^2 + Q^2\sigma_{12}\sigma_{21}}}, \tag{A.8}$$

$$\frac{\partial \Psi}{\partial \sigma_{22}} = \frac{\frac{1}{4}\cdot(N\sigma_{11} - P\sigma_{22})\cdot(-P)}{\sqrt{\frac{1}{4}\cdot(N\sigma_{11} - P\sigma_{22})^2 + Q^2\sigma_{12}\sigma_{21}}}, \tag{A.9}$$

$$\frac{\partial \Psi}{\partial \sigma_{12}} = \frac{Q^2\cdot\sigma_{21}}{2\cdot\sqrt{\frac{1}{4}\cdot(N\sigma_{11} - P\sigma_{22})^2 + Q^2\sigma_{12}\sigma_{21}}}, \tag{A.10}$$

$$\frac{\partial \Psi}{\partial \sigma_{21}} = \frac{Q^2\cdot\sigma_{12}}{2\cdot\sqrt{\frac{1}{4}\cdot(N\sigma_{11} - P\sigma_{22})^2 + Q^2\sigma_{12}\sigma_{21}}}, \tag{A.11}$$

$$\frac{\partial \Lambda}{\partial \sigma_{11}} = \frac{\frac{1}{4} \cdot (R\sigma_{11} - S\sigma_{22}) \cdot R}{\sqrt{\frac{1}{4} \cdot (R\sigma_{11} - S\sigma_{22})^2 + T^2\sigma_{12}\sigma_{21}}}, \tag{A.12}$$

$$\frac{\partial \Lambda}{\partial \sigma_{22}} = \frac{\frac{1}{4} \cdot (R\sigma_{11} - S\sigma_{22}) \cdot (-S)}{\sqrt{\frac{1}{4} \cdot (R\sigma_{11} - S\sigma_{22})^2 + T^2\sigma_{12}\sigma_{21}}}, \tag{A.13}$$

$$\frac{\partial \Lambda}{\partial \sigma_{12}} = \frac{T^2 \cdot \sigma_{21}}{2 \cdot \sqrt{\frac{1}{4} \cdot (R\sigma_{11} - S\sigma_{22})^2 + T^2\sigma_{12}\sigma_{21}}}, \tag{A.14}$$

$$\frac{\partial \Lambda}{\partial \sigma_{21}} = \frac{T^2 \cdot \sigma_{12}}{2 \cdot \sqrt{\frac{1}{4} \cdot (R\sigma_{11} - S\sigma_{22})^2 + T^2\sigma_{12}\sigma_{21}}}. \tag{A.15}$$

Alternatively, one may calculate the yield function gradient more conveniently by means of the forward difference scheme (Aretz, 2003) as follows:

$$\frac{\partial F}{\partial \sigma_{11}} \approx \frac{F(\sigma_{11} + \Delta\sigma, \sigma_{22}, \sigma_{12}, \sigma_{21}) - F(\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{21})}{\Delta\sigma}, \tag{A.16}$$

$$\frac{\partial F}{\partial \sigma_{22}} \approx \frac{F(\sigma_{11}, \sigma_{22} + \Delta\sigma, \sigma_{12}, \sigma_{21}) - F(\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{21})}{\Delta\sigma}, \tag{A.17}$$

$$\frac{\partial F}{\partial \sigma_{12}} \approx \frac{F(\sigma_{11}, \sigma_{22}, \sigma_{12} + \Delta\sigma, \sigma_{21}) - F(\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{21})}{\Delta\sigma}, \tag{A.18}$$

$$\frac{\partial F}{\partial \sigma_{21}} \approx \frac{F(\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{21} + \Delta\sigma) - F(\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{21})}{\Delta\sigma}. \tag{A.19}$$

A central difference scheme, which is computationally more expensive, has also been formulated (Aretz, 2003) but there is no practical difference in the accuracy. $\Delta\sigma$ is of the order 1×10^{-5}. If necessary one should symmetrize the gradient calculated in this numerical fashion since

$$\frac{\partial F}{\partial \sigma_{12}} = \frac{\partial F}{\partial \sigma_{21}} \tag{A.20}$$

must hold.

References

Alt, W., 2002. Nichtlineare Optimierung. Vieweg-Verlag.

Aretz, H., 2003. Modellierung des anisotropen Materialverhaltens von Blechen mit Hilfe der Finite-Elemente-Methode. Ph.D. Thesis, RWTH Aachen University, Shaker-Verlag.

Banabic, D., Bunge, H.-J., Pöhlandt, K., Tekkaya, A.E., 2000a. Formability of Metallic Materials. Springer-Verlag.

Banabic, D., Comsa, S.D., Balan, T., 2000b. A new yield criterion for anisotropic sheet metals under plane stress conditions. In: Proceedings of the Cold Metal Forming 2000 Conference, Cluj Napoca, pp. 217–224.

Barlat, F., Richmond, O., 1987. Prediction of tricomponent plane stress yield surfaces and associated flow and failure behaviour of strongly textured F.C.C. polycrystalline sheets. Mat. Sci. Eng. 91, 15–29.

Barlat, F., Lian, J., 1989. Plastic behaviour and stretchability of sheet metals – Part I: A yield function for orthotropic sheets under plane stress condition. Int. J. Plasticity 5, 51–66.

Barlat, F., Lege, D.J., Brem, J.C., 1991. A six-component yield function for anisotropic materials. Int. J. Plasticity 7, 693–712.

Barlat, F., Becker, R.C., Hayashida, Y., Maeda, Y., Yanagawa, M., Chung, K., Brem, J.C., Lege, D.J., Matsui, K., Murtha, S.J., Hattori, S., 1997a. Yielding description for solution strengthened aluminum alloys. Int. J. Plasticity 13, 385–401.

Barlat, F., Maeda, Y., Chung, K., Yanagawa, M., Brem, J.C., Hayashida, Y., Lege, D.J., Matsui, K., Murtha, S.J., Hattori, S., Becker, R.C., Makosey, S., 1997b. Yield function development for aluminium alloy sheets. J. Mech. Phys. Solids 45, 1727–1763.

Barlat, F., Banabic, D., Cazacu, O., 2002. Anisotropy in Sheet Metals. In: Proceedings of the NUMISHEET 2002 Conference, Jeju Island, pp. 515–524.

Barlat, F., Brem, J.C., Yoon, J.W., Chung, K., Dick, R.E., Choi, S.-H., Pourboghrat, F., Chu, E., Lege, D.J., 2003. Plane stress yield function foraluminium alloy sheets – part 1: theory. Int. J. Plasticity 19, 1297–1319.

Bassani, J.L., 1977. Yield characterisation of metals with transversally isotropic plastic properties. Int. J. Mech. Sci. 19, 651–654.

Bron, F., Besson, J., 2004. A yield function for anisotropic materials – Application to aluminum alloys. Int. J. Plasticity 20, 937–963.

Budiansky, B., 1984. Anisotropic plasticity of plane-isotropic sheets. In: Dvorak, G.J., Shield, R.T. (Eds.), Mechanics of Material Behaviour. Elsevier, Amsterdam, pp. 15–29.

Cazacu, O., Barlat, F., 2001. Generalization of Drucker's yield criterion to orthotropy. Math. Mech. Solids 6, 613–630.

Gotoh, M., 1977. A theory of plastic anisotropy based on a yield function of fourth order. Int. J. Mech. Sci. 19, 505–520.

Hill, R., 1948. A theory of the yielding and plastic flow of anisotropic metals. Proc. R. Soc. Lond. A 193, 281–297.

Hill, R., 1979. Theoretical plasticity of textured aggregates. Math. Proc. Cambridge Philos. Soc. 85, 179–191.

Hill, R., 1990. Constitutive modeling of orthotropic plasticity in sheet metals. J. Mech. Phys. Solids 38, 405–417.

Hill, R., 1993. A user-friendly theory of orthotropic plasticity in sheet metals. Int. J. Mech. Sci. 15, 19–25.

Hill, R., 1998. The Mathematical Theory of Plasticity. Clarendon Press (reprint).

Hosford, W.F., 1972. A generalized isotropic yield criterion. J. Appl. Mech. 39, 607–609.

Karafillis, A.P., Boyce, M.C., 1993. A general anisotropic yield criterion using bounds and a transformation weighting tensor. J. Mech. Phys. Solids 41, 1859–1886.

Kreißig, R., 1981. Theoretische und experimentelle Untersuchungen zur plastischen Anisotropie. Ph.D. Thesis, TU Karl-Marx-Stadt.

Müller, W., 1996. Beitrag zur Charakterisierung von Blechwerkstoffen unter zweiachsiger Beanspruchung. Ph.D. Thesis, University of Stuttgart, Springer-Verlag.

Paraianu, L., Comsa, D.S., Cosovici, G., Jurco, P., Banabic, D., 2003. An improvement of the BBC2000 yield criterion. In: Proceedings of the ESAFORM 2003 Conference.

Press, W.H., 1992. Numerical Recipes in C – The Art of Scientific Computing. Cambridge University Press.

Sallat, G., 1988. Theoretische und experimentelle Untersuchungen zum Fließverhalten von Blechen im zweiachsigen Hauptspannungszustand. Ph.D. Thesis, TU Karl-Marx-Stadt.

Stoughton, T.B., Yoon, J.-W., 2004. A pressure-sensitive yield criterion under a non-associated flow rule for sheet metal forming. Int. J. Plasticity 20, 705–731.

Tourki, Z., Makkouk, R., Zeghloul, A., Ferron, G., 1994. Orthotropic plasticity in metal sheets. J. Mater. Process. Technol. 45, 453–458.
Vegter, H., Drent, P., Huetink, J., 1995. A planar isotropic yield criterion based on mechanical testing at multi-axial stress states. In: Proceedings of the NUMIFORM 1995 Conference, pp. 345–350.