

Earing Prediction in Cup Drawing using the BBC2008 Yield Criterion

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Abstract. The paper deals with constitutive modelling of highly anisotropic sheet metals. It presents FEM based earing predictions in cup drawing simulation of highly anisotropic aluminium alloys where more than four ears occur. For that purpose the BBC2008 yield criterion, which is a plane-stress yield criterion formulated in the form of a finite series, is used. Thus defined criterion can be expanded to retain more or less terms, depending on the amount of given experimental data. In order to use the model in sheet metal forming simulations we have implemented it in a general purpose finite element code ABAQUS/Explicit via VUMAT subroutine, considering alternatively eight or sixteen parameters (8p and 16p version). For the integration of the constitutive model the explicit NICE (Next Increment Corrects Error) integration scheme has been used. Due to the scheme effectiveness the CPU time consumption for a simulation is comparable to the time consumption of built-in constitutive models. Two aluminium alloys, namely AA5042-H2 and AA2090-T3, have been used for a validation of the model. For both alloys the parameters of the BBC2008 model have been identified with a developed numerical procedure, based on a minimization of the developed cost function. For both materials, the predictions of the BBC2008 model prove to be in very good agreement with the experimental results. The flexibility and the accuracy of the model together with the identification and integration procedure guarantee the applicability of the BBC2008 yield criterion in industrial applications.

Keywords: Anisotropy, Yield criteria, BBC2008 model, Earing prediction, Numerical integration, NICE scheme, Simulation.

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INTRODUCTION

Numerical simulation of the manufacturing processes is an important way of reducing the overall production costs and the failure rate. During the last decades, this computational instrument has extended its applicability in various industrial fields. The earing phenomenon has been noticed in the early applications of the rolled metallic sheets for the deep-drawing of cylindrical parts. The number of ears is usually four, but there are also situations when six or eight ears may occur at the upper edge of the drawn cups. Which particular evolution of the earing process will take place depends generally on the anisotropy of the sheet metal and the lubrication conditions.

The development of the numerical techniques during the last three decades allows the earing prediction by finite element simulation. The quantitative analyses related to the accuracy of the numerical results show that the constitutive models used in the simulation have a significant influence on the predicted earing profile. As a consequence, many researchers have focused their interest on the investigation of the possibilities to improve the quality of the numerical predictions by adopting more realistic material models. The anisotropic yield criterion proposed by Hill in 1948 [1] was extensively used for the simulation of the cup drawing processes with the aim of analyzing the earing profile. But, since earing prediction based on the Hill 1948 yield criterion proved to be rather limited, more sophisticated yield criteria have been developed [2-14] during the last two decades. Some of these material models have been implemented in commercial finite element codes and used for a prediction of the earing profile.

As mentioned above, cylindrical cups obtained by deep-drawing usually exhibit four ears. In the case of some materials having a more specific anisotropy, six or even eight ears may develop during the drawing process. The experimental and theoretical studies have proved the existence of direct relationship between the number of ears and the variation of the anisotropy coefficient in the plane of the sheet metal. The occurrence of more than four ears can be predicted only by those yield criteria that use at least eight material parameters associated to different planar directions. In fact, such plasticity models have been developed mainly as a response to this challenge. An analysis of the number of mechanical parameters needed to correspondingly describe the anisotropy and their influence on the predicted earing profile has been performed by Soare and Banabic [15] using the Poly 8 yield criterion. The first yield criterion that uses general stress state and more than eight mechanical characteristics in the identification procedure has been developed by Barlat et al. [7]. The capability of this model to predict the occurrence of six or eight ears in the drawing of cylindrical cups has been proved by Yoon et al. in a series of papers [16-21]. Aiming at improving the accuracy of the earing predictions Yoon et al. [20] have taken in their simulations also the evolution of the anisotropy during the forming process into account.

Comsa and Banabic [22] have recently proposed a simple plane-stress yield criterion (BBC 2008) defined as an extension of the BBC 2005 model [12]. As an alternative of using linear transformations of the stress tensor and absolute value function, the model is very convenient for finite element implementation. Due to the fact that in the identification procedure of the new constitutive model more than eight mechanical parameters can be used, a more accurate description of the anisotropy is possible. The capability of the BBC 2008 yield criterion to predict the occurrence of more than four ears will be proved in the next sections of this paper by performing corresponding numerical simulations of the cylindrical cup deep-drawing with two aluminium alloys considered.

BBC 2008 YIELD CRITERION

The BBC 2008 yield criterion is a plane-stress criterion developed to describe the plastic behaviour of highly orthotropic sheet metals [22]. The yield surface defined by this model results from the implicit equation

$$\Phi(\sigma_{11}, \sigma_{22}, \sigma_{12} = \sigma_{21}, Y) := \bar{\sigma}(\sigma_{11}, \sigma_{22}, \sigma_{12} = \sigma_{21}) - Y = 0 \quad (1)$$

where $\bar{\sigma}(\sigma_{11}, \sigma_{22}, \sigma_{12} = \sigma_{21}) \geq 0$ is the equivalent stress (see below), $Y > 0$ is the yield stress, while σ_{11} , σ_{22} , and $\sigma_{12} = \sigma_{21}$ are planar components of the stress tensor expressed in an orthonormal basis superimposed to the local axes of plastic orthotropy. One assumes that the third unit vector of the local basis is always perpendicular to the mid-surface of the sheet metal.

In the particular case of the BBC 2008 yield criterion, the equivalent stress is defined as follows [22]:

$$\begin{aligned} \frac{\bar{\sigma}^{2k}}{w-1} &= \sum_{r=1}^s \left\{ w^{r-1} \left[\left[L^{(r)} + M^{(r)} \right]^{2k} + \left[L^{(r)} - M^{(r)} \right]^{2k} \right\} + w^{s-r} \left\{ \left[M^{(r)} + N^{(r)} \right]^{2k} + \left[M^{(r)} - N^{(r)} \right]^{2k} \right\}, \\ L^{(r)} &= \ell_1^{(r)} \sigma_{11} + \ell_2^{(r)} \sigma_{22}, \\ M^{(r)} &= \sqrt{\left[m_1^{(r)} \sigma_{11} - m_2^{(r)} \sigma_{22} \right]^2 + \left[m_3^{(r)} (\sigma_{12} + \sigma_{21}) \right]^2}, \\ N^{(r)} &= \sqrt{\left[n_1^{(r)} \sigma_{11} - n_2^{(r)} \sigma_{22} \right]^2 + \left[n_3^{(r)} (\sigma_{12} + \sigma_{21}) \right]^2}, \\ k, s &\in \mathbb{N} \setminus \{0\}, \quad w = (3/2)^{1/s} > 1, \quad \ell_1^{(r)}, \ell_2^{(r)}, m_1^{(r)}, m_2^{(r)}, m_3^{(r)}, n_1^{(r)}, n_2^{(r)}, n_3^{(r)} \in \mathbb{R}. \end{aligned} \quad (2)$$

The quantities denoted as k , $\ell_1^{(r)}$, $\ell_2^{(r)}$, $m_1^{(r)}$, $m_2^{(r)}$, $m_3^{(r)}$, $n_1^{(r)}$, $n_2^{(r)}$ and $n_3^{(r)}$ ($r=1, \dots, s$) are material parameters. It is easily noticeable that the equivalent stress defined by Eq. (2) reduces to the isotropic formulation proposed by Barlat and Richmond [23] if

$$\ell_1^{(r)} = \ell_2^{(r)} = m_1^{(r)} = m_2^{(r)} = m_3^{(r)} = n_1^{(r)} = n_2^{(r)} = n_3^{(r)} = 1/2, \quad r = 1, \dots, s. \quad (3)$$

Under these circumstances, the value of the integer exponent k may be adopted according to the crystallographic structure of the sheet metal, as in Barlat and Richmond's model: $k = 3$ for BCC alloys, and $k = 4$ for FCC alloys.

The other material parameters involved in Eq. (2) are evaluated upon a corresponding identification procedure. Their number is $n_p = 8s$, where $s \in \mathbb{N} \setminus \{0\}$ is the summation limit. Let n_e be the number of experimental values describing the plastic anisotropy of the sheet metal. If $n_e \geq 8$, the summation limit s must be chosen according to the constraint $s \leq n_e / 8$. When $n_e < 8$, the minimum value $s = 1$ must be adopted. In this case, the identification

constraints obtained from experiments should be accompanied by at least $8 - n_e$ artificial conditions involving the material parameters. For example, if $n_e = 6$, one may enforce $m_1^{(1)} = n_1^{(1)}$ and $m_2^{(1)} = n_2^{(1)}$.

Due to the expandable structure of the yield criterion, many identification strategies can be devised. Comsa and Banabic developed a numerical procedure based on the minimisation of an error-function operating only with normalized yield stresses and r-coefficients obtained from uniaxial / biaxial tensile tests [22].

IMPLEMENTATION OF THE BBC 2008 YIELD CRITERION IN ABAQUS

The above presented constitutive model has been implemented in a general purpose finite element code ABAQUS via VUMAT subroutine. For the integration of the constitutive model the NICE (*Next Increment Corrects Error*) explicit integration scheme, developed recently by some co-authors, is used. Its task is to find a proper increment of the plastic multiplier $\Delta\lambda$ from a given total strain increment $\Delta\varepsilon_{ij}$. The basic ideas of the NICE scheme are presented in Halilovic et al. [24], whereas in Vrh et al. [25] the reader can find its theoretical background adequately elaborated. The comparison studies and the proof given in the appendix of the latter paper show, that the accuracy of the new scheme is comparable to the accuracy of the backward-Euler scheme, while it is computationally far more (up to ten times) efficient in explicit dynamics simulations.

Implementation of the constitutive model via user subroutine requires integration of the constitutive model along a known strain path, which is mathematically reflected in known total strain increments $\Delta\varepsilon_{ij}$. Although deduction of the NICE integration scheme is general, its implementation for shell applications needs a particular care. Namely, in order to satisfy the zero normal stress condition during the whole integration a through-thickness strain increment has to be adequately chosen in each integration step.

Treatment of zero normal stress constraint in shell applications

General approach for calculation of the through-thickness strain increment is derived in Vrh et al [25]. In classical elasto-plastic plane stress applications the user of the NICE scheme can calculate the through-thickness strain increment with the following simple equation

$$\Delta\varepsilon_{33} = - \frac{C_{33kl} \Delta\varepsilon_{kl}^* - \left(\Phi + \frac{\partial\Phi}{\partial\sigma_{ij}} C_{ijkl} \Delta\varepsilon_{kl}^* \right) \Theta \beta}{\left(C_{3333} - \Theta^2 \beta \right)} \quad (4)$$

where

$$\beta = \left(\frac{\partial\Phi}{\partial\sigma_{ij}} C_{ijkl} \frac{\partial\Phi}{\partial\sigma_{kl}} - \frac{\partial\Phi}{\partial Y} \frac{\partial Y}{\partial\varepsilon_{eq}^p} \frac{\partial\Phi}{\partial\sigma_{ij}} \frac{\sigma_{ij}}{Y} \right)^{-1}, \quad \Theta = \frac{\partial\Phi}{\partial\sigma_{ii}} C_{ii33}, \quad \varepsilon_{ij}^* = \begin{cases} 0 & ; i = j = 3 \\ \varepsilon_{ij} & ; \text{otherwise} \end{cases} \quad (5)$$

and Φ is the plastic potential (see Eq. (13)). In the above equations derivative $\partial\Phi/\partial\sigma_{33}$ is required which defines the influence of the through-thickness stress on yielding. The most elegant way to overcome this enigma is upgrading of the yield criterion with a simple substitution of the stress state using the deviatoric expression

$$\Phi(\sigma_{11}, \sigma_{22}, \sigma_{12}, Y) = \Phi(\sigma_{11} - \sigma_{33}, \sigma_{22} - \sigma_{33}, \sigma_{12}, Y) \quad (6)$$

Note that with such a manipulation the plane stress problem solution is not influenced since $\sigma_{33} = 0$, whereas the plastic incompressibility condition

$$\partial\Phi/\partial\sigma_{11} + \partial\Phi/\partial\sigma_{22} + \partial\Phi/\partial\sigma_{33} = 0 \quad (7)$$

is preserved.

Integration of BBC 2008 with NICE scheme

Assuming, that in each increment the total strain increments are available from the computed increments of the displacement field (note that strain increments $\Delta\varepsilon_{kl}^*$ are then given as input in VUMAT subroutine for further

processing, whereas $\Delta\varepsilon_{33}$ can be calculated by considering (4)), the stress state at the end of the increment must be calculated using the following system of differential and algebraic equations:

$$\Phi(\sigma_{ij}, Y) = 0 \quad (8)$$

$$\begin{aligned} \Delta\sigma_{ij} &= C_{ijkl} (\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^p) \\ \Delta\varepsilon_{ij}^p &= \frac{\partial\Phi}{\partial\sigma_{ij}} \Delta\lambda \\ \Delta\varepsilon_{eq}^p &= \frac{\sigma_{kl}}{Y} \frac{\partial\Phi}{\partial\sigma_{kl}} \Delta\lambda \\ \Delta Y &= \frac{\partial Y}{\partial\varepsilon_{eq}^p} \Delta\varepsilon_{eq}^p \end{aligned} \quad (9)$$

The plastic deformation is constrained by the consistency condition $\Phi = 0$, Eq. 8, which must be respected through all the integration process during the evolution of plastic strains. According to the NICE scheme, this is achieved by expanding the consistency condition into Taylor series, where higher order differentials are neglected. The numerical scheme is thus based, provided the values of the state variables are known at the beginning of the considered increment, on imposing

$$\Phi + d\Phi = 0 \quad (10)$$

to be fulfilled in the considered increment, which leads to

$$\Phi + \frac{\partial\Phi}{\partial\sigma_{ij}} \Delta\sigma_{ij} + \frac{\partial\Phi}{\partial Y} \Delta Y = 0 \quad (11)$$

With regard to the forward-Euler approach which uses a differential form of the consistency condition, i.e. $d\Phi = 0$, our approach considers the additional term Φ . Though this term should be zero, because it represents a function whose value should obey the consistency condition $\Phi = 0$, numerically this is usually not true. This small difference between the two explicit schemes, NICE and forward-Euler, is the key reason for a considerable improvement in stability of the numerical integration.

The remaining equations in the above DAE system, which accomplish the integration procedure, are the evolution equations, here expressed in the incremental form (9). Considering those equations and expanded consistency condition (11) yields the increment of the plastic multiplier

$$\begin{aligned} \Delta\lambda &= \frac{\Phi + \frac{\partial\Phi}{\partial\sigma_{ij}} C_{ijkl} \Delta\varepsilon_{kl}}{\frac{\partial\Phi}{\partial\sigma_{ij}} C_{ijkl} \frac{\partial\Phi}{\partial\sigma_{kl}} - \frac{\partial\Phi}{\partial Y} \frac{dY}{d\varepsilon_{eq}^p} \frac{\sigma_{kl}}{Y} \frac{\partial\Phi}{\partial\sigma_{kl}}} \end{aligned} \quad (12)$$

With the increment of the plastic multiplier $\Delta\lambda$ calculated the incremental solution of the elasto-plastic model, defined by the evolution equations (9), can be considered solved.

The numerical integration procedure requires a calculation of the yield function Φ and its respective derivatives in every increment. For implementation purposes, the yield function can be rewritten in the following form:

$$\Phi = \frac{\bar{\sigma}^{2k}(\sigma_{ij})}{Y^{2k}} - 1 = \frac{\hat{\sigma}(\sigma_{ij})}{Y^{2k}} - 1 = 0 \quad (13)$$

with the derivatives used in the numerical scheme being:

$$\begin{aligned} \frac{\partial\Phi}{\partial Y} &= -2k \frac{\hat{\sigma}}{Y^{2k+1}}; \quad \frac{\partial\Phi}{\partial\hat{\sigma}} = \frac{1}{Y^{2k}} \\ \frac{\partial\Phi}{\partial\sigma_{ij}} &= \frac{\partial\Phi}{\partial\hat{\sigma}} \sum_{r=1}^s \left[\frac{\partial\hat{\sigma}}{\partial L^{(r)}} \frac{\partial L^{(r)}}{\partial\sigma_{ij}} + \frac{\partial\sigma}{\partial M^{(r)}} \frac{\partial M^{(r)}}{\partial\sigma_{ij}} + \frac{\partial\hat{\sigma}}{\partial N^{(r)}} \frac{\partial N^{(r)}}{\partial\sigma_{ij}} \right] \end{aligned} \quad (14)$$

where

$$\begin{aligned}
\frac{\partial \hat{\sigma}}{\partial L^{(r)}} &= 2k(w-1)w^{r-1} \left[\left(L^{(r)} + M^{(r)} \right)^{2k-1} + \left(L^{(r)} - M^{(r)} \right)^{2k-1} \right] \\
\frac{\partial \hat{\sigma}}{\partial N^{(r)}} &= 2k(w-1)w^{s-r} \left[\left(M^{(r)} + N^{(r)} \right)^{2k-1} - \left(M^{(r)} - N^{(r)} \right)^{2k-1} \right] \\
\frac{\partial \hat{\sigma}}{\partial M^{(r)}} &= 2k(w-1) \left\{ \begin{aligned} &w^{r-1} \left[\left(L^{(r)} + M^{(r)} \right)^{2k-1} - \left(L^{(r)} - M^{(r)} \right)^{2k-1} \right] + \\ &w^{s-r} \left[\left(M^{(r)} + N^{(r)} \right)^{2k-1} + \left(M^{(r)} - N^{(r)} \right)^{2k-1} \right] \end{aligned} \right\}
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
\frac{\partial L^{(r)}}{\partial \boldsymbol{\sigma}} &= \left\{ \ell_1^{(r)}, \ell_2^{(r)}, -(\ell_1^{(r)} + \ell_2^{(r)}), \mathbf{0} \right\} \\
\frac{\partial M^{(r)}}{\partial \boldsymbol{\sigma}} &= \frac{1}{M^{(r)}} \left\{ \tilde{M}^{(r)} m_1^{(r)}, -\tilde{M}^{(r)} m_2^{(r)}, \tilde{M}^{(r)} (m_2^{(r)} - m_1^{(r)}), m_3^{(r)} 2\sigma_{12} \right\} \\
\frac{\partial N^{(r)}}{\partial \boldsymbol{\sigma}} &= \frac{1}{N^{(r)}} \left\{ \tilde{N}^{(r)} n_1^{(r)}, -\tilde{N}^{(r)} n_2^{(r)}, \tilde{N}^{(r)} (n_2^{(r)} - n_1^{(r)}), n_3^{(r)} 2\sigma_{12} \right\} \\
\tilde{M}^{(r)} &= m_1^{(r)} (\sigma_{11} - \sigma_{33}) - m_2^{(r)} (\sigma_{22} - \sigma_{33}) \\
\tilde{N}^{(r)} &= n_1^{(r)} (\sigma_{11} - \sigma_{33}) - n_2^{(r)} (\sigma_{22} - \sigma_{33})
\end{aligned} \tag{16}$$

Equation (16) is due to simplicity written in Voigt notation, where $\frac{\partial \square^{(r)}}{\partial \boldsymbol{\sigma}} = \left\{ \frac{\partial \square^{(r)}}{\partial \sigma_{11}}, \frac{\partial \square^{(r)}}{\partial \sigma_{22}}, \frac{\partial \square^{(r)}}{\partial \sigma_{33}}, \frac{\partial \square^{(r)}}{\partial \sigma_{12}} \right\}$ for

$r = 1, \dots, s$.

Let us remind that according to the explicit approach all the state variables appearing in the above equations and expressions are written at the beginning of the considered increment. Once the increment of the plastic multiplier $\Delta\lambda$ is calculated, the respective increments of the other state variables can be readily calculated using Equations (9). The integration procedure is presented in Figure 1.

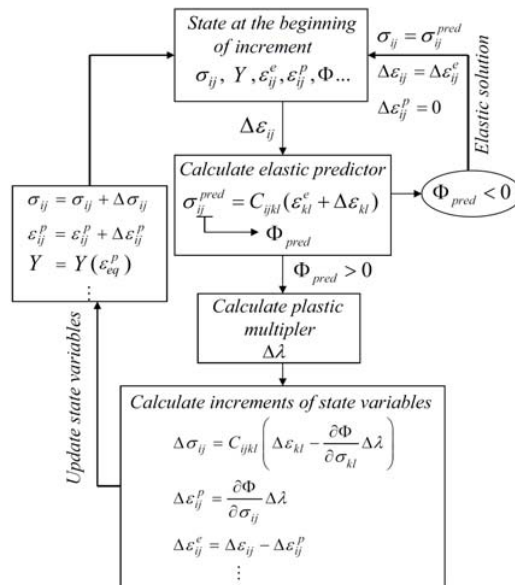


FIGURE 1. Integration (implementation) of constitutive model

EARING PREDICTION USING THE BBC2008 YIELD CRITERION

In this section a validation of the presented approach is discussed, based on two performed round cup deep drawing simulations. Due to orthotropic material model only a quarter section of the cup with the corresponding symmetric boundary conditions applied is analyzed. A total of 2560 shell elements with reduced integration (ABAQUS S4R) and 21 section points through the sheet thickness are used for the simulation. To model the elastoplastic sheet metal response the BBC2008 model is employed. Its implementation with respective parameters identified for each of the sheet metals considered is enabled via user material subroutine VUMAT. The material parameters of the equivalent stress were calculated using experimental values of the normalized yield stresses and r-coefficients in uniaxial and biaxial tension. A detailed description of the BBC2008 identification procedure is given in [22].

Earing prediction for AA5042-H2

The simulation of deep drawing of aluminium AA5042-H2 follows the one, presented in Yoon et al. [20]. The tool geometry is as follows: blank diameter is 76.07mm, die opening diameter is 46.74 mm, punch diameter is 45.72 mm, die-profile radius and punch-profile radius are 2.28 mm. The initial thickness of the blank is 0.274 mm and holder force is 10 kN. Displacement of the punch is set to be large enough to pull the whole blank into the die. The friction is considered with the Coulomb model, the coefficient of friction being 0.008 for all contact surfaces. In this case the Voce hardening law $Y = 404.16 - 107.17 e^{-18.416 \epsilon_{eq}^p}$ is assumed to model the work hardening behaviour of the sheet metal. Material data (normalized yield stresses and r-coefficients) are given in [20], whereas the identified parameters of the BBC2008 model are given in Table 1.

TABLE 1. Parameters for 16 parameters BBC2008 version for AA5042-H2

k	s	w	$\ell_1^{(1)}$	$\ell_2^{(1)}$	$m_1^{(1)}$	$m_2^{(1)}$	$m_3^{(1)}$
4	2	1.224744	0.719915	0.350253	0.573905	0.606047	0.718855×10^{-4}
$n_1^{(1)}$	$n_2^{(1)}$	$n_3^{(1)}$	$l_1^{(2)}$	$l_2^{(2)}$	$m_1^{(2)}$	$m_2^{(2)}$	$m_3^{(2)}$
0.306974	0.492406	0.15355×10^{-5}	0.564817	0.322344	0.374385	0.468435	0.737118
$n_1^{(2)}$	$n_2^{(2)}$	$n_3^{(2)}$					
0.320818	0.641638	0.					

Figure 2a shows the final geometry of deep drawn cup, as simulated, with the equivalent plastic strain field distribution displayed, whereas in Figure 2b a comparison between the predicted ears and the experimental ones [20] is given where $h(\alpha)$ denotes the height of the deep drawn cup as a function of angle α (angle from the rolling direction). Also, for the sake of comparison, the calculated ears profile with the CPB06ex2 model [20] is included.

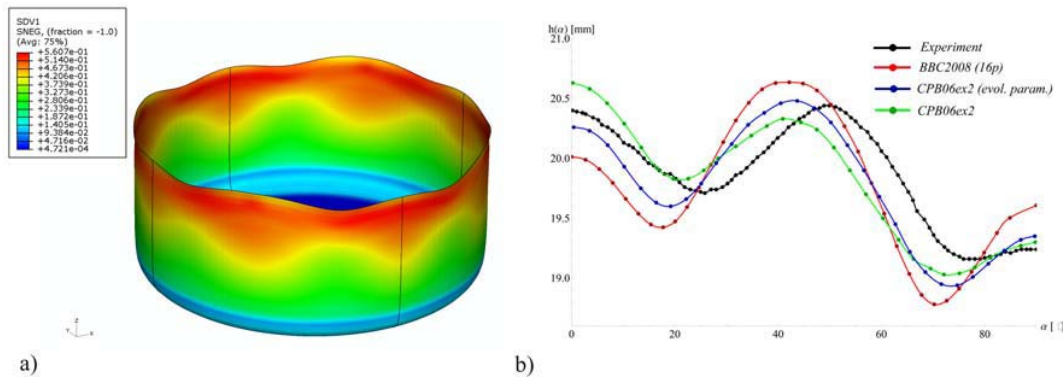


FIGURE 2. Earing prediction for aluminium AA5042-H2, a) Simulation b) Ears profile

Prediction of ears profile with the BBC2008 model is in good agreement with results of *CPB06ex2* model, proposed by Yoon et al. [20]. The predicted ears are in same location, but are predicted to be little more intense. Eight ears are predicted by the model and agreement with experimental results is, at least qualitatively, very good.

Earing prediction for AA2090-T3

BBC2008 is in this section validated also on aluminium AA2090-T3. The round cup drawing experiment and a simulation with Yld2004 model is presented in Yoon et al. [18]. In this case the tool geometry is as follows: blank diameter is 158.76 mm, die opening diameter is 101.48 mm, punch diameter is 97.46 mm, die-profile radius and punch-profile radius are 12.70 mm. The initial thickness of blank is 1.6 mm and holder force is 22.2 kN. The friction is considered with the Coulomb model, the coefficient of friction being 0.1 for all contact surfaces. Hardening is modelled with the following stress-strain curve $Y = 646 \left(0.025 + \varepsilon_{eq}^p \right)^{0.227}$. The identified parameters of the BBC2008 model for 8 and 16 parameters versions are given in Table 2 and Table 3, respectively.

TABLE 2. Parameters for 8 parameters BBC2008 version for AA2090-T3

k	s	w	$\ell_1^{(1)}$	$\ell_2^{(1)}$	$m_1^{(1)}$	$m_2^{(1)}$	$m_3^{(1)}$	$n_1^{(1)}$	$n_2^{(1)}$	$n_3^{(1)}$
4	1	1.500000	0.449938	0.513218	0.630315	0.601445	0.727299	0.153818	0.479391	0.499818

TABLE 3. Parameters for 16 parameters BBC2008 version for AA2090-T3

k	s	w	$\ell_1^{(1)}$	$\ell_2^{(1)}$	$m_1^{(1)}$	$m_2^{(1)}$	$m_3^{(1)}$	$n_1^{(1)}$	$n_2^{(1)}$
4	2	1.22474	0.130866	0.621742	0.783422	0.660402	0.79×10^{-4}	0.110991	0.048245
$n_3^{(1)}$	$l_1^{(2)}$	$l_2^{(2)}$	$m_1^{(2)}$	$m_2^{(2)}$	$m_3^{(2)}$	$n_1^{(2)}$	$n_2^{(2)}$	$n_3^{(2)}$	
0.307522	1.033922	-0.071963	1.13×10^{-4}	0.77×10^{-4}	0.538047	0.055764	1.018603	0.778150	

Figure 3a displays the simulated final geometry of deep drawn cup (for 16 parameters model) and the corresponding equivalent plastic strain distribution, whereas in Figure 3b a comparison between predicted and experimental ears is given. Also, for the sake of comparison, ears profile calculated with the Yld2004 model [18] is included.

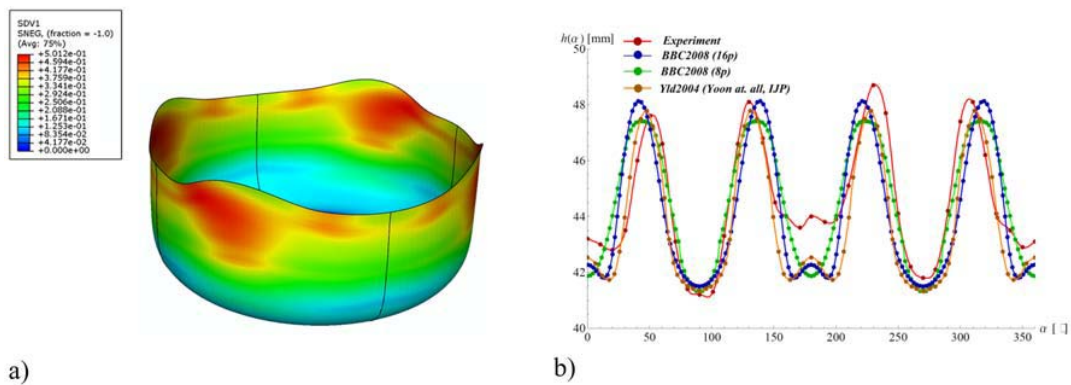


FIGURE 3. Earing prediction for aluminium AA2090-T3, a) Simulation b) Ears profile

Predictions of the BBC2008 model are in good agreement with a prediction of the Yld2004 model and also with the experiment [18]. As expected, the 8 parameters version was unable to predict six ears, which were experimentally observed. On the contrary, the 16 parameters version predicts 6 ears and their location, and at least qualitatively, the results are in good agreement with the experiment.

CONCLUSION

In the paper the authors have shown an approach to modelling of plastic anisotropy in numerical simulation, which includes the BBC2008 plane-stress yield criterion, an efficient procedure for its implementation and a procedure for the identification of the respective model parameters. The constitutive model is derived in a form of a finite series that can be correspondingly expanded to retain more or less terms, depending on the available experimental data. The simulations of deep drawing of a round cup, which were performed with ABAQUS/Explicit with shell S4R elements, prove the capability of the model to predict more than four ears occurrence. Further, considering a comparison with the experimentally obtained ears profiles we can conclude that the presented approach is physically objective and is able to predict complex plastic anisotropic behaviour of sheet metal. Since the BBC2008 model does not use linear transformations of the stress tensor and absolute value function in the yield criterion formulation it is very convenient for implementation in finite element programs. Due to the fact, that it is in this paper explicitly integrated, the computational efficiency of the approach should be superior in the simulation of sheet metal forming processes comparing to existing techniques.

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